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Location-routing problem with fuzzy time windows and traffic time

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Abstract

The location-routing problem is a relatively new branch of logistics system. Its objective is to determine a suitable location for constructing distribution warehouses and proper transportation routing from warehouse to the customer. In this study, the location-routing problem is investigated with considering fuzzy servicing time window for each customer. Another important issue in this regard is the existence of congested times during the service time and distributing goods to the customer. This caused a delay in providing service for customer and imposed additional costs to distribution system. Thus we have provided a mathematical model for designing optimal distributing system. Since the vehicle location-routing problem is Np-hard, thus a solution method using genetic meta-heuristic algorithm was developed and the optimal sequence of servicing for the vehicle and optimal location for the warehouses were determined through an example.

Keywords: Locating Routing; fuzzy time window; satisfaction level; congested times

1. Introduction

Today, rising energy costs and increasing competition have forced logistics sector to improve the efficiency of transportation network. Network design is a fundamental step in designing an effective supply chain. There are many different decisions in distribution network design that consists of determining the optimal location of facilities of supply chain. These decisions are generally categorized in strategic, tactical and operational levels. Strategic or long-term decisions require high investment and have vital importance for firms in order to survive among competitors. Tactical

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decisions consist of midterm decisions. Operational decisions mostly include tasks scheduling that are performed regularly (Fazel-Zarandi et al., 2013). Location routing problem as a relatively new branch of logistics system, includes strategic and tactical decisions. It consists of two major and associated parts including the facility location activities at the strategic level and determining the routing structure at the tactical level. In many practical situations, a combined location-routing model, such as done by Prodhon (2010) has been investigated. In that, by Wright and Clark method, is proposed a suitable model for a variety of LRP problems by combining the routing problem and facility location problem based on algorithms of random development. The location-routing problem can be often solved separately and only recent works have solved these two problems at the same time. Simultaneous solving of location and routing problem is a challenging task and will be beneficial for Logistics management and supply chain decision makers (Derbel et al., 2012). To solve these models, first the conceptual structure of LRP is considered and then we search optimal location of facilities and routing design of system (Tavakkoli-Moghaddam and Makuib, 2010). This means that LRP is similar to location- allocation (LAP). But the LAP does not consider tours when the facility is located. If warehouse places are fixed and the only objective is to find the optimal routing between warehouses and customers, then LRP becomes VRP. In recent decades LRP has been more investigated. In a recent review study, Nagy and Salehi (2007) classified different location-routing models and their assumptions. They also categorized different methods for LRP solving. location-routing problems can be categorized according to these characteristics consists of hierarchical layers, structural levels, number of the facilities, fleet size, transportation capacity, facility capacity, demand, planning, time windows, number of objective functions, the solution space, data types and methods for solving. This paper continues as follows: In the next section, the literature review of location routing problem with time window is presented. In Section 3, the model and its mathematical formulation is described. Section 4 discusses the solution analysis and numerical results come in Section 5. We conclude the paper in section 6.

2. Literature review

Various techniques are emerged while investigating the studies and researches done in LRP field. The main methods are categorized as (1) Exact methods, (2) Heuristic methods and (3) Meta-heuristic approaches. Exact methods search among all possible scenarios and choose the best one. This causes that the scale of solvable problems does not exceed a certain limit, thus, this method is not suitable for large scale problems delete does not work. In this regard, we can refer to the research of Contardo et al. (2012) which tried to solve the capacitated location-routing problem, combining branch and cut and neighborhood search methods. Max and Lian (2007) proposed a non-linear integer programming model for stochastic supply chain design problem in which Lagrange multipliers relaxation is used. Other solving methods are known as classic method and heuristic method that are structured to find near optimal answers. The third class of the methods of this kind is called meta-heuristic methods which explore the solution space of problems to find desired solutions. Extensive research is written on heuristic and meta-heuristic techniques to solve location-routing problems with warehouse capacity constraint or vehicles constraint or both constrains. Javid et al. (2010) has considered location-routing decisions, warehouse capacity decisions in a stochastic supply chain system. Their model in large scale is solved via combining methods of simulated annealing and tabu search. Derbel et al. (2010) have used genetic algorithm and local search to solve their location-routing model. Based on their findings, the hybrid algorithm has better results compared to tabu search method. Ting and Chen (2013) divided location-routing problem into two sub-problems of location and routing, and they used an ant colony method to solve the model. Caballero et al. (2007) has suggested a model for localizing where to construct two ovens for destroying animal waste and routing for offering services to different slaughter houses across Spain and has used the tabu search to achieve scientific findings. The model presented by Sibel and Kara-Bahar (2007) focuses on the objectives of minimizing the cost and risk in transportation of dangerous waste in Turkey. Another study on transporting dangerous materials, suggested location-routing model in transportation networks of 20 states of the USA which include highways and railroads is done by Xie et al. (2012). Problem considered in this study has cost and risk constraints and mixed integer programming is used to solve it. Whenever a constraint is added to the problem, a new problem is raised such as situations when time window limitation is considered. According to the surveys, few papers have considered location-routing problem with time window and the rest of the papers have examined this concept in terms of vehicle routing problem (VRP). Vehicle routing problem time window (VRPTW) has been studied in theoretical researches and practical applications in the last 20 years (Prodhon, 2010). Table 1 shows an example of the work done in this context. Time windows for servicing are often encountered in Logistic practical problems so that each customer must be served in her/his own time window. Sexton and Choi(1986) were first to introduce time window into their model. Considering time window is strictly dependent on the customer satisfaction rate so that if a customer service is delayed compared to his/her desired time, it will lead to his/her dissatisfaction. Although this deviation from time window does not often cause any monetary penalty, fluctuations of customer satisfaction leads to damaging the benefits in the long term. In the conducted studies, the time window has often been considered certain. Fazel-Zarandi et al. (2011) consider a capacitated vehicle location-routing problem using fuzzy transportation time and certain time window for supplying the customers' demands. What has to be taken into account regarding time window, is that the customer may demand time window less than what is needed, or that satisfying all time windows lead to inflexibility or ineffability of solution. Therefore, this problem is absolutely dependent on customer behavior and, random events. Therefore, the time window is highly uncertain, stochastic and associated with human emotion. For example, given time by a customer may be expressed in the form of phrases like "about 9 o'clock". This approximation and lack of adequate information has led that we use fuzzy logic to formulate time windows in the model. This theory was first suggested by Zadeh (1965). Paper by Wang & Wan (2002) is the first study written regarding routing network using fuzzy theory for postman problem with taking into account the time windows. In the studies done, applying fuzzy time window to model vehicle location-routing has not been widely addressed and only Fazel Zarandi et al. (2013) have attempted to model and solve this problem. That is why there is a need for further research on this field. This research having considered the model in an uncertain situation has taken into account travel time and demand and presents fuzzy variables in validity situation. This study aims at, based on the failure experienced during solving the model, minimizing the total cost of travel package and the facility location and also minimizing the extra travel distance. This extra distance happens when the vehicle fails on serving some customers while on its route. The current study intends to increase customer satisfaction in services, based on the fuzzy demands of the customers while trying to minimize the costs. What makes this paper different from other studies in this field is that it attempts at making the conditions more real for vehicle location-routing problem in model. What is worthy of note here is the high-traffic routes while offering services and distributing the products to the customers in certain times of the day. This fact causes some problems in offering services to the customers in requested time windows that causes delay in offering those services and finally leads to customer satisfaction. It also brings about difficulties in unloading. On the other hand, extra costs (time and fuel consumption) are imposed to the system. Therefore, suggesting a solution in this regard can influence product distribution so that it can be designed in a more appropriate approach and prevent extra costs and customer dissatisfaction. Hence, the current study investigates vehicle capacitated location-routing problem in fuzzy demand situation via taking into account fuzzy time window in order to offer services to the customers and through looking at the traffic density of the transportation routes. We did not find any study dealing with this important issue in the literature.

Table 1. Overview of the work done in the logistics field with regard to time window

Solution Method	Style of problem
GLNPSO-EP	VRPSTW
A two-stage algorithm	VRPFTW
Goal programming and genetic algorithms	VRPTW
Generated column	VRPSSTW
Goal programming	VRPTW
genetic algorithms and DEA	VRPTW
tabu search	VRPTW
Hybrid intelligent algorithms (fuzzy simulation and genetic)	VRPTW
Integer programming	LRPTW
Simulated annealing	LRPFTW

3. Mathematical models

The model presented in this paper, offers a solution for the location - routing problem with fuzzy time window and includes objectives such as minimizing total costs and maximizing customer satisfaction level. It is assumed that the capacity of each vehicle and each customer's demand is constant and known. Each vehicle route starts from the distribution center and ends at supply centers. Travel time is considered to be uncertain and fuzzy. Vehicles are the same and each customer only receives service (delete serves) from one vehicle. Only one warehouse provides all services of one client requests.

3.1. Level of customer satisfaction

In the traditional time windows, each client request must be met in the specified time frame. So if the customer is serviced in specified time frame it will be acceptable and the satisfaction level is good but customer satisfaction levels with minimal distortion of time window is reduced to zero in these cases.

This is known as a classic and binary definition of time window that are known as crisp time window. However, in real life, a small deviation of time window is accepted and the level of customer satisfaction is not zero in these cases. Thus for a certain time window, two following concepts are introduced for each client (Xu et al., 2011).

Endurable earliness time (EET): the earliest service time that a customer can endure when a service starts earlier than e.
Endurable lateness time (ELT): the latest service time that a customer can endure when a service starts later than l.
In this case, we have formed a fuzzy window. A diagram of such time windows is shown in Figure 1. In this figure, parameters e and l are limits of crisp time window that provides the highest utility for the customer.

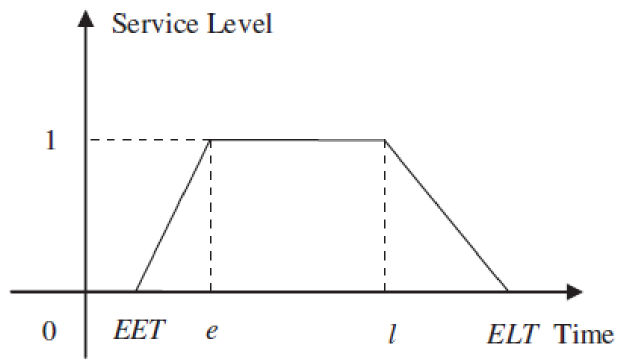

Figure 1. outline window fuzzy

So, in this case, the customer satisfaction levels is not good and bad (0 or 1) and depending on time of service gets the values between zero and one.

$$L(t) = \begin{cases} 0 & t < EET \\ \dfrac{t - EET}{e - EET} & EET \leq t \leq e \\ 1 & e \leq t \leq l \\ \dfrac{ELT - t}{ELT - l} & l < t \leq ELT \\ 0 & t > ELT \end{cases}$$

Where $\frac{t- EET}{e-EET}$ is a non-descending (Ascending) function of t with values between zero and one and $\frac{ELT-t}{ELT-l}$ is descending function of t. The utility level of service can be described at various service times using function$L(t)$.

3.2.The traffic density in solving the model

In a network with multiple customers and multiple routes, traffic load is not distributed uniformly. Accordingly, at one time, some of the routes are crowded, and the traffic is low in the other routes. In this study, to avoid servicing at the traffic time, for certain customers who are located in traffic routes, special time windows are considered that it will prevent providing the service for customer at traffic time. This time window is defined in the interval$[G , GG]$ where G and GG are beginning and end of the traffic time in the traffic route. These times can be provided by traffic police and traffic control centers. In this model, N is the node set that contains: i = 1, .., m potential warehouses and j = m +1, ..., n customers. There is a transportation cost between the nodes (i, j) that is C_{ij} so that C_{ij}=C_{ji}. Warehouse i has a capacity equal to w_i and d_j is the demand of customer j. The number of vehicles is limited to k and the capacity of each one is q. $\tilde{t}_{ij}$ is travel time between two nodes i and j that are considered as the triangular fuzzy numbers and we assume $\tilde{t}_{ij}$=$\tilde{t}_{ji}$. T_j is the servicing time of node j when the tour is obtained. $[EET_j, E_j, L_j, ELT_j]$ is fuzzy time window of each customer. The goals include: increasing levels of customer satisfaction and minimizing the total cost. The total cost includes the warehouse cost (O_i), transportation cost (C_{ij}) and the vehicle fixed costs that are associated with each warehouse (F_i). In this problem, the number of customers is M. Variable y_i is a binary variable associate with the warehouse i where its value is equal to one if the warehouse i has been used otherwise it is zero. Variables associated with connecting node i to j with vehicle k, is x_{ijk}. If there is a connection between two nodes i and j then its value is equal one and it is zero otherwise. Finally, if warehouse i is connected to customer j then variable f_{ij} is one otherwise it is zero. Based on what was mentioned, the problem is formulated as follows. In this model, function (1) is used for maximizing the satisfaction level of each client. The constraint (12), (13) and (14) are used to consider nonlinear objective function (1) in the model consequently the linear objective function (3) has been obtained.

$$\max \left(\min\left(1, \frac{T_j - EET_j}{E_j - EET_j}, \frac{ELT_j - T_j}{ELT_j - L_j}\right)\right) \tag{1}$$

Objective functions:

$$\min \quad \sum_1^m O_i y_i + \sum_{i\in v}\sum_{j\in v}\sum_{k\in K} C_{ij} * x_{ijk} + \sum_{i\in v}\sum_{j\in v}\sum_{k\in K} F_i * x_{ijk} + E_j * p_j \tag{2}$$

$$\max \quad \frac{1}{M}\left(\sum_{j\in J} LL_j\right) \tag{3}$$

Constraints of the model are as follows:

$$\sum_{k\in K}\sum_{i\in v} x_{ijk} = 1 \qquad \forall\, j \in J \tag{4}$$

$$\sum_{j\in J}\sum_{i\in v} d_j * x_{ijk} \le q \qquad \forall k \in K \tag{5}$$

$$\sum_{j\in J} d_j * f_{ij} \le w_i * y_i \qquad \forall i \in I \tag{6}$$

$$\sum_{j\in v} x_{ijk} - \sum_{j\in v} x_{jik} = 0 \qquad \forall i \in v, k \in K \tag{7}$$

$$\sum_{k\in K}\sum_{i\in v} x_{ijk} \le 1 \qquad \forall j \in J \tag{8}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijk} \le |S| - 1 \qquad \forall S \subset j, k \in K \tag{9}$$

$$\sum_{u\in J} x_{iuk} + \sum_{u\in v\setminus\{j\}} x_{ujk} \le 1 + f_{ij} \qquad \forall i \in I, j \in J, k \in K \tag{10}$$

$$\sum_{k\in K}\sum_{i\in v} x_{ijk}\left(T_i + \tilde{t}_{ij}\right) = T_j \qquad \forall j \in J \tag{11}$$

$$(T_j - EET_j)/(E_j - EET_j)) \ge LL_j \qquad \forall j \in J \tag{12}$$

$$((ELT_j - T_j)/(ELT_j - L_j)) \geq LL_j \qquad \forall j \in J \tag{13}$$

$$0 \leq LL_j \leq 1 \qquad \forall j \in J \tag{14}$$

$$EET_j \leq T_j \leq ELT_j \qquad \forall j \in J \tag{15}$$

$$p_j > \varepsilon * (G - T_j) * (T_j - GG) \qquad \forall j \in J \tag{16}$$

$$f_{ij} \in \{0, 1\} \qquad \forall i \in I, j \in J \tag{17}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall i \in I, j \in J, k \in K \tag{18}$$

$$y_i \in \{0, 1\} \qquad \forall i \in I \tag{19}$$

$$P_j \in \{0, 1\} \qquad \forall j \in J \tag{20}$$

In the mathematical model, equation (2) represents the first objective function which minimizes the transportation total cost and the warehouses construction cost. In this regard, the total cost includes of (delete) the warehouse cost, the vehicle fixed cost, the transportation cost from one node to another node and resulted penalty cost from exposure to the time window of traffic routes. The second objective function, equation (3), maximizes average of customer satisfaction level. Equation (4) denotes that each customer is assigned to only one route. Equation (5) expresses that the total customer demand on a route does not exceed the vehicle capacity. Equation (6) denotes that the total goods supplied from any used warehouse should not exceed its capacity. Equation (7) expresses that if a vehicle entered into each node, then it must exit from that node. Equation (8) denotes that a customer demand is supplied from a one (delete) warehouse. Equation (9) is added to omit sub tour where S is a subset of the customer nodes. Equation (10) ensures that the client must be connected to the warehouse if there is a path between them. This constraint implies that the vehicle which exits from the warehouse, then it comes back to it. Equation (11) expressed the vehicle's arrival time to each customer according to the tour route sequence. Equations (12), (13) and (14) are added to the model in order to consider objective function of the level of the customer satisfaction as linear function. Equation (15) is based on the sequence selection where the service time per customer's request should be placed within the time window. Equation (16) gives a penalty to the objective function if the vehicle is located in traffic time window. In this regard, $[G, GG]$is time window associated with crowded Hours. Equations (17), (18), (19) and (20) indicates the decision variables types. LRP is an NP-hard problem because it is formed of two NP-hard problems (location of facilities and vehicle routing) (Fazel-Zarandi et al., 2011). Therefore exact methods cannot be effective for solving large problems and heuristic and Meta-heuristic methods must be used to solve. A genetic algorithm (GA) is used in order to solve the proposed model. Therefore, due to the dependence of the minimum and maximum values simultaneously and the requirement to use near optimal solution, techniques such as multi-objective utility function (MAUT) or methods of fuzzy programming do not seem to be suitable for converting multiple objectives into a single one. We investigated methods such as multi-objective utility function and it is observed that its consequences did not show good convergence process. Therefore, the weighted sum method is used to integrate the objective functions. This method is so easy to understand for decision maker and allow him to examine the objectives based on the targets importance by giving different weight to each one.

Therefore, U_i is defined as the weight for objective i. Conflicting goals can be aligned with a negative coefficient.

$$\max \quad U(x) = \sum U_i\big(f_i(x)\big) \tag{21}$$

4. Genetic Algorithm

Due to the nature of NP-hard problem, one approach for problem solving is to use Meta-heuristics algorithms, thus, genetic algorithm has been chosen to solve the proposed model. Genetic algorithm is a powerful algorithm for solving optimization problems and engineering designs (Stanciulescu et al., 2003). In this paper, first we present a simulated algorithm for calculation purposes then solution structure in terms of genetic algorithms for solving the location - routing problem with fuzzy time windows and traffic hours is described.

4.1.The proposed algorithm for solving the model

Since the travel time between the warehouse and the customer is uncertain, the triangular fuzzy numbers are defuzzified, in order to transform the fuzzy problem into an equivalent crisp problem. This method is summarized as follows according to Fazel Zarandi et al. (2013). For each customer, a number t is generated in the interval between upper and lower limit of triangular fuzzy numbers. Next a random number β is generated in the range of zero and one. λ is the membership function for value of t. As long as, λ is smaller than random number β, Then the value of t and λ will be accepted. In the other words, when $\beta < \boldsymbol{\lambda}$ then value of t is reported as the simulation time. Other steps of algorithm are as follows,

Step 1: as an initial solution, arrange all clients and warehouses in a list of numbers (string). Figure 2 is an example of a string that includes 3 warehouses and 7 customers. In this string, numbers 1 to 3 are warehouses and numbers 4 to 10 are customers.

5	2	4	7	1	9	6	3	8	10

Figure 2. Illustration of a string with 3 warehouses and 7 customers

step2: The first warehouse in the list is open, and the customers are allocated to the warehouse according to a sequence shown in Figure 2 until the full capacity of the warehouse. In this step, customers' demands continue until the full capacity of the vehicle. Whenever a vehicle is fully loaded, another vehicle is used. Also in this step, the time of offering a service to the customers is calculated during the allocation and based on the time of that service on each part of the time window, customer satisfaction level regarding the time of receiving the service is determined. Also, if the time of offering a service is delayed compared with the last expected time for offering that service, the customer will be still on the list of unallocated customers. Meanwhile, if the considered customer is among the special customers in terms of traffic condition, the time of providing a service for them in time window of the traffic period is examined and if it belongs to traffic period then a penalty cost will be added to the current costs. .

Step 3: Clients who are examined in the previous step will be removed from the list of unallocated customers.

Step 4: If warehouse capacity constraint is violated, the warehouse will be removed from the initial list. In this case, Return to step 2 again, and the remaining unallocated clients are assigned to the next warehouse.
Step 5: If any client is still remained from the primary list back to step 2.
Step 6: with allocating all clients, the algorithm finishes.

4.1.1. Genetic Algorithm

Step 1: Initial population generation: a genetic algorithm starts with an initial population of solutions. Each solution is displayed by a chromosome that is a string of bits. All possible solutions should be displayed using a coding system.
Step 2: Determine the fitness value after the customer's allocation to warehouses, according to mentioned steps in the proposed algorithm, the fitness level of each chromosome is determined.
Step 3: Populations Generating
Selection: The first step involves selecting parent from population for generation of new solutions. This chosen selection is done randomly with regards to a probability that is proportional to their fitness function. At this stage, it should be decided about how to select parents for crossover operation, how to generate offspring and numbers of children. Parents are often chosen by using roulette wheel method. In this paper, we used this method too.
Generation: In the second step, Recombination and mutation operators on selected individuals are used and new chromosomes are generated.
Crossover: single-point crossover operator is the most common crossover operator. This operator is used in this research. In this operation, two chromosomes are broken up in one point randomly and broken parts of two chromosomes are displaced with each other. Thus, two new chromosomes are obtained. The initial chromosomes are parent chromosomes and chromosome resulted from the displacement action are child. See Figure 3.
Mutation: After crossover, Chromosomes change with mutation operator. Mutation operator will prevent the algorithm falling into local optimum. In this mutation, two locations that were selected randomly are displaced with each other. Figure 4 shows some of these mutations.
Step 4: Stopping rule: the algorithm stops when the population converges to the optimal solution or near optimal solution. In this study, the maximum generation and no improvement in the fitness is used as a stopping criterion.

Figure 3. crossover operator in genetic algorithms

Figure 4. mutation operator in genetic algorithms

5. Numerical results

In this section, we present an example to elaborate how the algorithm works. In this example, 4 warehouses are potentially intended to serve 15 clients; the warehouses have different costs and different capacities. Warehouses are numbered with number 1 to 4 and clients are denoted with the numbers 5 to 19. Transportation time between each clients and their warehouses are in the form of triangular fuzzy numbers (a, b, c), are shown in Table 2. Table 3 shows the transportation cost between the clients and warehouse. Clients demand and time window for servicing clients are shown by trapezoidal fuzzy numbers $(a, a + \alpha * a, d - d * \alpha, d)$, which are presented in Table 4. The capacity of each vehicle is 600 units. In this example, for the customers with number of 5, 7, 8,11,12,15 and 17, traffic time interval is defined. If the vehicle enters in this time interval, then we are faced with 1,000 unit penalty cost. The data used in this study, has been adapted from (Zheng and Liu, (2006)), but customer time window and traffic time windows are added to this example. The program was codified using Matlab and was run in a 2-GIG memory PC. The results (with 100 ring repetitions and 100 production sequences as chromosome in each ring) exhibit the optimal plan for vehicles' mobility as below. The results show that optimized design for moving vehicles is as follows. Best parameters of genetic algorithm, is obtained by using Taguchi analysis based on the values in Table 5 and are obtained as follows. $\alpha = 0.3$, crossover rate=0.5, mutation rate=0.2, where the optimal value of objective function is equal 5146.611. For integrating objective functions, a minimization objective function is considered. So the minimum values shown in Figure 5 are considered as the most desirable result for the chosen parameters.

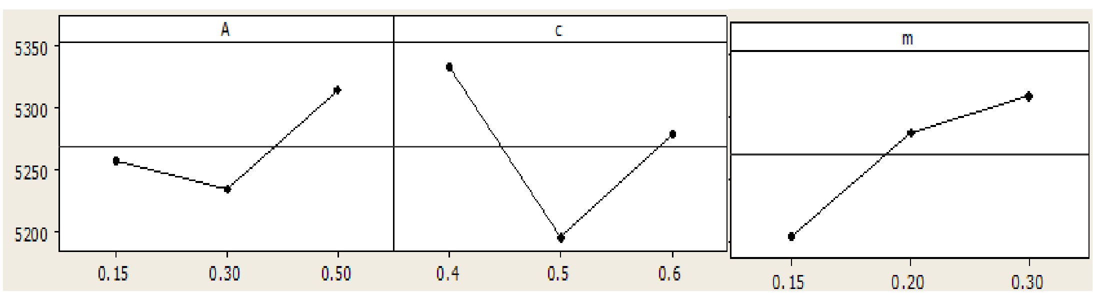

Figure 5. Experimental design based on Taguchi analysis for parameters α, crossover and mutation rates (left to right)

The obtained results are as follows;

- Only 2 warehouses from 4 warehouses are opened.
- The vehicles are assigned as follows

Vehicle1 (warehouse 1): 6 » 7 » 8 » 10

Vehicle 2 (the warehouse 1): 5 » 9

Vehicle 3 (the warehouse 2): 11 » 12 » 13 » 14 » 15

Vehicle 4 (the warehouse 2): 17 » 16 » 18 » 1

Table 2. Transportation time matrix

Number	1	2	3	4	5	6	7
1							
2	(5, 10, 15)						
3	(25, 50, 75)	(5, 10, 15)					
4	(7, 15, 23)	(25, 50, 75)	(7, 15, 23)				
5	(25, 50, 75)	(17, 35, 53)	(17, 35, 53)	(15, 30, 45)			
6	(25, 50, 75)	(7, 15, 23)	(20, 40, 60)	(2, 5, 8)	(22, 45, 68)		
7	(12, 25, 38)	(20, 40, 60)	(15, 30, 45)	(17, 35, 53)	(7, 15, 23)	(12, 25, 38)	
8	(7, 15, 23)	(20, 40, 60)	(5, 10, 15)	(22, 45, 68)	(10, 20, 30)	(17, 35, 53)	(20, 40, 60)
9	(25, 50, 75)	(7, 15, 23)	(22, 45, 68)	(5, 10, 15)	(22, 45, 68)	(15, 30, 45)	(5, 10, 15)
10	(10, 20, 30)	(22, 45, 68)	(12, 25, 38)	(22, 45, 68)	(7, 15, 23)	(15, 30, 45)	(20, 40, 60)
11	(25, 50, 75)	(5, 10, 15)	(17, 35, 53)	(15, 30, 45)	(17, 35, 53)	(5, 10, 15)	(15, 30, 45)
12	(27, 55, 83)	(17, 35, 53)	(17, 35, 53)	(15, 30, 45)	(17, 35, 53)	(2, 5, 8)	(15, 30, 45)
13	(5, 10, 15)	(20, 40, 60)	(5, 10, 15)	(20, 40, 60)	(7, 15, 23)	(15, 30, 45)	(17, 35, 53)
14	(25, 50, 75)	(5, 10, 15)	(20, 40, 60)	(2, 5, 8)	(22, 45, 68)	(15, 30, 45)	(2, 5, 8)
15	(22, 45, 68)	(5, 10, 15)	(20, 40, 60)	(5, 10, 15)	(22, 45, 68)	(15, 30, 45)	(5, 10, 15)
16	(7, 15, 23)	(22, 45, 68)	(7, 15, 23)	(22, 45, 68)	(10, 20, 30)	(15, 30, 45)	(22, 45, 68)
17	(15, 30, 45)	(20, 40, 60)	(12, 25, 38)	(20, 40, 60)	(10, 20, 30)	(12, 25, 38)	(17, 35, 53)
18	(25, 50, 75)	(5, 10, 15)	(22, 45, 68)	(5, 10, 15)	(25, 50, 75)	(15, 30, 45)	(7, 15, 23)
19	(25, 50, 75)	(20, 40, 60)	(20, 40, 60)	(22, 45, 68)	(17, 35, 53)	(15, 30, 45)	(17, 35, 53)

8	9	10	11	12	13	14
8						
9	(20, 40, 60)					
10	(5, 10, 15)	(12, 25, 38)				
11	(5, 10, 15)	(17, 35, 53)	(17, 35, 53)			
12	(7, 15, 23)	(17, 35, 53)	(17, 35, 53)	(12, 25, 38)		
13	(17, 35, 53)	(5, 10, 15)	(20, 40, 60)	(20, 40, 60)	(17, 35, 53)	
14	(17, 35, 53)	(17, 35, 53)	(5, 10, 15)	(20, 40, 60)	(15, 30, 45)	(15, 30, 45)
15	(17, 35, 53)	(17, 35, 53)	(2, 5, 8)	(20, 40, 60)	(15, 30, 45)	(20, 40, 60)	(20, 40, 60)
16	(17, 35, 53)	(10, 20, 30)	(22, 45, 68)	(15, 30, 45)	(17, 35, 53)	(7, 15, 23)	(7, 15, 23)
17	(2, 5, 8)	(12, 25, 38)	(20, 40, 60)	(5, 10, 15)	(12, 25, 38)	(12, 25, 38)	(12, 25, 38)
18	(17, 35, 53)	(20, 40, 60)	(7, 15, 23)	(20, 40, 60)	(15, 30, 45)	(20, 40, 60)	(20, 40, 60)
19	(17, 35, 53)	(20, 40, 60)	(22, 45, 68)	(15, 30, 45)	(7, 15, 23)	(17, 35, 53)	(20, 40, 60)

15	16	17	18	19
15				
16	(22, 45, 68)			
17	(17, 35, 53)	(17, 35, 53)		
18	(7, 15, 23)	(7, 15, 23)	(20, 40, 60)	
19	(2, 5, 8)	(22, 45, 68)	(12, 25, 38)	(20, 40, 60)

Table 3. The Transportation cost between the clients and warehouses

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0																		
2	19	0																	
3	17.5	6	0																
4	28	11	10.5	0															
5	24	21	15	20	0														
6	24.5	32	26	34	15.5	0													
7	31.2	44.5	39.5	49	31	16	0												
8	31	48.5	45	55.5	41.5	28.5	16	0											
9	21	37.5	33.5	44	30	18.5	13	11.5	0										
10	18	36	33	44	38	24	20	13.5	7	0									
11	21.5	40	39	49.5	43	36.5	32.5	21	20	13	0								
12	36.5	55	54	65	56	46	37	21	28	23	15.5	0							
13	31.5	46.5	48	57	55.5	51	48.5	36	36	29	16	21.5	0						
14	23	38.5	39	48.5	47	44	43	32	30	23	12	23	9	0					
15	28	38.5	41.5	49	52	51	52.5	43.5	40	33	22.5	33	13	11	0				
16	34.5	40	44	50	56.5	58.5	62	54	50	43	34	44.5	24	22	11.5	0			
17	30	29.5	34.5	38	48.5	54	60.5	56	49	43.5	38	51.5	33	28	20.5	14	0		
18	18.5	16.5	21.3	26.5	35	41	50	48	39	35	33	48	34	27	24	23.5	13.5	0	
19	24	14	20	22	35	44	54.5	55	45	41.5	41	56.5	43	35	32	36	17	8.5	0

Table 4. Demand, expected time window, and the customer traffic time interval

number	demand	expected time window	Traffic interval	number	demand	expected time window	Traffic interval
5	160	[9:22, 11:09, 12:05, 12:17]	[9:40, 10:10]	13	200	[9:15, 9:19, 10:59, 11:50]	
6	200	[9:12, 9:15, 11:09, 12.05]		14	80	[9:12, 9:15, 11:09, 12.05]	
7	60	[9:12, 9:15, 11:09, 12.05]	[9:20, 9:50]	15	60	[9:12, 9:15, 11:04, 11.57]	[10:50, 11:25]
8	200	[9:12, 9:15, 10:59, 11.50]	[9:18, 9:45]	16	200	[9:12, 9:15, 11:04, 11.57]	
9	135	[9:13, 9:16, 11:04, 11:57]		17	90	[9:12, 9:15, 11:38, 12.47]	[10:25, 10:40]
10	120	[9:12, 9:15, 11:09, 12.05]		18	200	[9:15, 9:19, 11:09, 12:05]	
11	140	[9:15, 9:19, 10:04, 11:57]	[9:30, 10:00]	19	100	[9:15, 9:19, 11:02, 11:55]	
12	100	[9:12, 9:15, 11:15, 12.13]	[10:10, 10:35]				

- Based on the above sequence, the customer service time in each tour is as follows

Vehicle1 (warehouse 1): 35 » 62 » 85 » 94

Vehicle 2 (the warehouse 1): 38 » 65

Vehicle 3 (the warehouse 2): 6 » 23 » 43 » 61 » 82

Vehicle 4 (the warehouse 2): 15 » 36 » 46 » 75

We Assume, the start time of the service is 9 am, which is equal to 0. So the results show that customer service times are not in traffic hours.

6. Results Validation

In order to validate the results, we show that how our robust algorithm works by using various algorithm parameters.
Table 5 shows sensitivity analysis results on various parameters used in the model. In this Table, p_c represents the crossover rate, p_e represent mutation rate and α represents the change percentage in the range of customer's time window used in trapezoidal fuzzy numbers$(a, a + \alpha * a, d - d * \alpha, d)$. Based on different values for above parameters, optimum solution has been reported. Finally, error terms are reported. The amount of percent error is obtained from following equation.

$$\text{percent error} = \frac{\text{actual value} - \text{optimal value}}{\text{optimal value}} * 100$$

The results show that the deviation percentage of objective function doesn't exceed than 4.5 percent. This shows that model has credibility and stability in the different situations. The proposed approach is effective to solve the problem considered in this paper.

Table 5. Results comparison with different values of the problem parameters.

pop size	α	p_e	p_c	Function value	error
100	0.15	0.2	0.6	5279.16	0.022751
100	0.15	0.15	0.4	5282.505	0.023407
100	0.15	0.3	0.5	5211.5	0.009651
100	0.3	0.2	0.5	5269.129	0.020816
100	0.3	0.3	0.4	5341.54	0.034844
100	0.3	0.15	0.6	5161.685	0
100	0.5	0.3	0.6	5396.845	0.045559
100	0.5	0.2	0.4	5377.814	0.041872
100	0.5	0.15	0.5	5170.208	0.001651

7. Conclusion

In this paper, we provided a location-routing model with fuzzy time windows in terms of travel time uncertainty with regard to traffic restrictions on congested routes. A genetic algorithm is used to solve the model. Model results are illustrated by a numerical example. The sensitivity of the optimum solution has been investigated by changing parameters affecting on the model. As future researches, the transportation time and demand may be considered to be non-deterministic. Also using other meta-heuristics algorithm is also suggested as future researches. It also obtained transportation costs in a more real form and based on the inflation rates and fluctuations in fuel price, considered reopening the warehouses taking into account its future benefits in the model. This study has considered traffic routes and times, so the research results can be applied in urban transportation system which the traffic routes and times are important parts of it.

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