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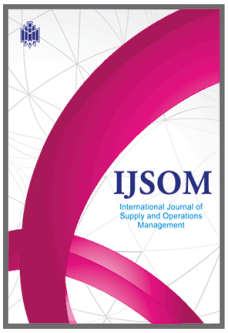

Modeling and numerical analysis of revenue sharing contract based on the Stackelberg game theory

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Abstract
Considered supply chain in this article consists of one vendor and multiple retailers where the vendor applies vendor managed inventory. Considering vendor as a leader and retailers as followers, Stackelberg game theory is applied for modeling and analyzing this system. A general mixed integer nonlinear model is developed which can optimize the performance of the system under revenue sharing contract, wholesale price contract and centralized structure. Based on this model, we numerically analyzed the performance of revenue sharing contract in the considered supply chain and four states for revenue sharing contract are analyzed at the end. Moreover, in each state, performance of the system under revenue sharing contract is compared with the performance of the system under wholesale price contract and centralized structure.

Keywords: vendor managed inventory; Stackelberg game; revenue sharing contract; wholesale price contract; centralized structure.

1. Introduction

Vendor managed inventory (VMI) is a 'pull' replenishing system designed to enable Quick Response (QR) of the vendor to fluctuating demand. VMI represents the high level of partnership where the vendor is the primary decision-maker in order planning and inventory control. Under a VMI system, the supplier decides about the appropriate inventory levels of products and the appropriate inventory policies to maintain these levels (Tyan H. and Wee H., 2003). Thus, a VMI partnership has two main characteristics: (1) VMI mainly focuses on integrated inventory management by the vendor with the cooperation of his retailers, and (2) the vendor has the right to know his retailers' inventory and market information in order to implement VMI (Yu Y. et al., 2009).

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1.1 designs and analyzing of contracts in VMI system

Although VMI programs may bring benefits to participants, there are potential challenges in implementing VMI programs. For example, in a VMI program, the vendor is responsible for the retailer's inventory. As a result, the vendor suffers the cost of inventory management. Hence, the vendor may not be willing to participate in the program, if he does not want to bear the holding cost or the risk of excessive inventory. Some efforts on practicing VMI programs end up unsuccessfully where vendor suffers too much inventories and costs, and too frequent shipments (Guan R. and Zhao X., 2010). Therefore, designing and analyzing suitable contracts for improving VMI system performance and creating coordination among members of VMI strategy has become more and more important.

In the literature, academic researches on VMI programs mainly focus on the following three aspects: (1) investigating the benefits of VMI programs compared with normal supply models without VMI; (2) operational decisions in VMI programs; and (3) designing contracts for VMI programs (Guan R. and Zhao X., 2010).

In the first aspect of the VMI investigation, the benefits of VMI are usually compared with the traditional inventory control management. In this aspect, the research question is: "why we use vendor managed inventory?" For this stream of research we refer to Mishra B. K. and Raghunathan S., (2004), Yao Y. and Martin D., (2008), Wang C.X. (2009). The second research stream addresses the "how to implement Vendor managed inventory?" question. Especially, this stream related to the operational decisions in VMI programs. For this area, we refer to Yao Y., Dong Y. and Dresner M. (2007), Nachiappan S P. and Jawahar N. (2007), Xu K. and Leung M T. (2009).

In recent years, contract design for VMI programs has become an important issue and several different contracts are proposed. Cachon gives an extensive review of typical types of contracts proposed by the related researches for supply chain coordination (Cachon G. P., 2003). The most commonly used contract is the wholesale price contract (WP contract). With the WP contract, the supplier charges the retailer a price for each item. It is well known that a WP contract may not coordinate the supply chain perfectly. A popular contract in practice is revenue sharing contract (RS contract), where the retailer agrees to give a percent of his revenue to the supplier (Guan R. and Zhao X ., 2010). Another popular contract is franchising contract, where the vendor charges the retailer an up-front fee to carry the goods (regardless of the stock level) and then sells the goods at a wholesale price to the retailer. The quantity discount model is a mechanism that allows a joint optimal-order quantity for the buyer and vendor. In fact, under this mechanism, the supplier induces the buyer to order the global optimal quantity by offering him a price discount (Giannoccaro I. and Pontrandolfo P., 2004). Different types of discount contract exist, i.e. all-unit and incremental model. In addition to these, there are several contracts that mainly pertain to the single period and newsboy problem such as, buyback, sales rebate and quantity flexibility contracts.

In general, there are two kinds of supply chain structures: centralized and decentralized supply chains. In the centralized supply chain structure, the supply chain operates on the basis of centrally made decisions. In the decentralized structure, each firm makes its own decisions, based on its own knowledge, almost regardless of the rest of the supply chain (Li S., Zhu Z. and Huang L., 2009). A coordination model in a decentralized chain has two main objectives (1) increasing

profit of a decentralized chain up to a centralized supply chain structure (Achieving channel coordination), and (2) sharing the obtained benefits of the coordination model among the supply chain members to encourage all members to participate (Chaharsooghi K. and Heydari J., 2010). One of the main objectives of contract design is supply chain coordination, thus, a suitable contract must consider these objectives. A further important issue that must be considered in designing a contract is the so called win-win condition: this condition occurs when under the contract; each supply chain member obtains a profit higher than the amount he/she would get without contract. Otherwise, the supply chain actors would not be prompted to adopt the contract (Giannoccaro I. and Pontrandolfo P., 2004). In this paper, we consider these concerns in analyzing different contracts.

1.2. Revenue sharing contract (RS contract)

One type of the supply chain coordination contracts is RS contract. Based on the RS contract, the supplier charges cp per unit purchased and the retailer gives λ percent of his revenue to the supplier (it is also possible to design RS contract in which only regular revenue is shared)(Cachon G. P., 2003). Thus the RS contract includes two main parameters: cp and λ. Notice that if $\lambda=0$ the RS contract changes to the WP contract.

Many authors have addressed the effect and the role of RS contract in supply chain coordination. Cachon and Lariviere investigate RS contract in general and express the weaknesses of this contract in detail. They also compared the performance of this contract with the other contracts in a supply chain that consists of one vendor and one supplier (Cachon G.P., Lariviere M. A., 2005). Guan et al. used RS contract for supply chain coordination that consists of one vendor and one retailer while continuous review policy is used for controlling the inventory. They also compared RS contract with the franchising contract (Guan R. and Zhao X., 2010). Pen et al. considered a supply chain consisting of two manufacturers and one retailer wherein each of the manufacturers can choose WP contract or RS contract. They compared these two contracts under the different power structures in supply chain, then they analyzed a supply chain that includes one manufacturer and two retailers and compared the mentioned contracts (Pan K et al., 2010). Li et al. applied RS contract along with consignment contract in a supply chain that includes one manufacturer and one retailer, where a single-period product is produced and sold. They demonstrated that when the manufacturer and retailer are assumed to be risk-neutral under a very mild restriction on the demand distribution, the decentralized supply chain can be perfectly coordinated (Li S et al., 2009). Giannoccaro et al. consider a three stage supply chain which includes a manufacturer, a distributer and a retailer. A contract model based on the revenue sharing contract has been proposed to coordinate a three-stage supply chain. Their contract model was characterized by two different contracts: the first is offered by the distributer to the retailer and the second is offered by the manufacturer to the distributer (Giannoccaro I. and Pontrandolfo P., 2004). Sarathi et al. use RS and quantity discount contract for coordinating in a channel consisting of single retailer and single manufacturer. Demand in the retailer side is considered price and stock dependent. It was shown that combined contract improves the performance of the supply chain and win-win result for the both sides of the contract is ensured (Partha Sarathi G et al., 2014). Chen et.al consider a tow level supply chain as the following form: the upstream manufacturer produces a single product and sells it though a vertically separated retailer, under a

consignment contract with revenue sharing and slotting allowance. Demand is considered price and shelf-space sensitive and equilibrium analyses are carried out for centralized and decentralized settings with and without cooperation (Chen J.M. et al., 2011). Saha and Goyal study a two stage supply chain that is composed of one manufacturer and one retailer and demand of the product is price and stock dependent. Three coordinating contracts are proposed for channel coordination: (i) joint rebate contract (ii) wholesale rebate contract and (iii) cost sharing contract (Saha S. and Goyal.S.K., 2015). Desai studies the supply chain coordination using revenue-dependent sharing contract in the movie industry. It is shown that supply chain can be perfectly coordinated using both types of revenue sharing contracts. However, there exist situations in which revenue dependent contracts outperform revenue-independent contracts (Palsule-Desai. O.D, 2013).

In this paper, we extend the previous literature by considering the issue of revenue sharing and wholesale price contracts in a Vendor-Managed Inventory (VMI) model of a single-vendor and multiple-retailers where the demands at retailers' side are price-sensitive. We investigate and analyze different revenue sharing policies on the model based on wholesale price, cp, and the percent of the retailer's revenue to be paid to the vendor, λ . This system is analyzed based on the stackelberg game theory, where vendor is the leader and retailers are the followers. A general model is developed which can explain the performance of the system under revenue sharing contract, wholesale price contract and centralized structure. Based on this model, we numerically analyze the effect of revenue sharing contract in the considered supply chain. Moreover, performance of the system under revenue sharing contract compared with the system performance under wholesale price contract and centralized structure.

The rest of the paper is organized as follows: In Section 2, we introduce the concept of Stackelberg game and the related literature in this concept. Section 3 presents the notations and assumptions that are utilized in developing the proposed model. In Section 4, we derive the net profit functions of the vendor and his retailers. Section 5 presents the Stackelberg game model for the discussed VMI supply chain. In Section 6, sensitivity analysis of the system parameters is performed. In section 7, the effects of RS contract in VMI supply chain are analyzed. Section 8 presents an initiative approach to improve the performance of RS contract. Finally, section 9 concludes the paper.

2. Stackelberg game and its equilibrium

Game theory has become an essential tool in the analysis of supply chains with multiple agents, often with conflicting objectives. Game theory is a powerful tool for analyzing situations in which the decisions of multiple agents affect each agent's payoff. As such, game theory deals with interactive optimization problems (Cachon P. G. and Netessine S., 2003).

In a type of classification, games are classified to static and dynamic games. In the static games, players choose their strategies simultaneously, while in the dynamic games the players choose their strategies sequentially and each player chooses his strategies after decision making of the former player. The simplest possible dynamic game was introduced by Stackelberg. In a Stackelberg game model, player 1 chooses a strategy first (the Stackelberg leader) and then player 2 observes this decision and makes his own strategy choice (the Stackelberg follower). Since in many supply chain models the upstream firm (e.g. the wholesaler), possesses certain power over

the (typically smaller) downstream firm (e.g. the retailer), the Stackelberg equilibrium concept has found many applications in supply chain management literature (Cachon P. G. and Netessine S., 2003). The scenario where the supplier holds greater channel power is modeled as a Stackelberg game where the supplier is the leader and the retailers are followers. The equal-power scenario is modeled as a simultaneous-decision game (Bichescu.B.C. and Fry M.J., 2009).

The literature on the applications of game theory in supply chain management is fairly extensive. Cachon et al. surveyed extensively the applications of game theory in supply chain analysis. They discussed both non-cooperative and cooperative game in static and dynamic setting and used newsvendor model to demonstrate the applications of various tools (Cachon P. G. , Netessine S., 2003). Moreover, Nagarajan et al. and Fierstras et al. reviewed and analyzed the applications of game theory in supply chain management (Fiestras-Janeiro M.G. et al., (2010), Nagarajan M. and Sosic G. (2008)). Many authors have used Stackelberg game for analyzing the supply chain. Ygangue et al. used Stackelberg game concept for analyzing the VMI supply chain that consists of one manufacturer and multiple retailers, where the manufacturer is the leader of the game and the retailers are followers (Yu Y. et al., 2009, Yu Y. et al., 2009). Almehada et al. compare the VMI supply chain in two scenarios: leadership of the manufacturer versus the leadership of one of the retailers. They use Stackelberg game to analyze these two scenarios (Almehdawe E. and Mantin B., 2010). Beshesco and Fry formulated the situation wherein the supplier has more power, as the Stackelberg game and compare the centralized and decentralized supply chain, but they do not consider the VMI strategy (Bichescu.B.C. and Fry M.J., 2009). Qin et al. considered a one supplier-one retailer supply chain, and analyzed the system where demand is a decreasing function of price. They use Stackelberg game for analyzing this supply chain and applied quantity discount and franchising contract for supply chain coordination (Qin Y. et al., 2007). Braide and et al. analyzed a supply chain that consists of a single manufacturer and several heterogonous retailers, where the manufacturer is considered as the leader of the Stackelberg game. Demand is price sensitive and retailers are geographically dispersed, thus no competition exists between retailers (Braide S. et al., 2013).

In this paper, we consider the vendor as the leader of the supply chain, and formulate the system as the Stackleberg game. To find the Stackelberg equilibrium, we need to solve a dynamic two-period problem via backward induction: first, player 2 (follower) selects the best strategy by considering all possible strategies of the first player (leader) considering the best response of player 2, then, player 1 selects an appropriate strategy. If x_i and π_i are the selected strategy and payoff of player i, respectively, the Stackelberg equilibrium can be represented as follows (Cachon P. G. , Netessine S., 2003):

$$x_2^*(x_1): \frac{\partial \pi_2(x_2, x_1)}{\partial x_2} = 0,$$
$$\frac{\partial \pi_1(x_1, x_2^*(x_1))}{\partial x_1} = \frac{\partial \pi_1(x_1, x_2^*)}{\partial x_1} + \frac{\partial \pi_1(x_1, x_2)}{\partial x_2}\frac{\partial x_2^*}{\partial x_1} = 0 \quad (1)$$

In practice, is more convenient to apply the approach of Almehdawe and Mantin (Almehdawe E. and Mantin B. 2010) as follows:

Step 1- Formulate the followers' optimization problem

Step 2- Formulate the Leader's optimization problem

Step 3- Derive the Karush-Kuhn-Tucker (KKT) conditions for the followers' optimization problem
Step 4- Involve the KKT conditions in the leader's optimization problem.
The solution of final model in Step 4 gives the Stackelberg equilibrium. In this paper, we use this approach to find Stackelberg game equilibrium.
It should be noticed that none of the game players would like to deviate from the Stackelberg equilibrium point in order to maintain their profit. If the game leader wants to deviate from this point, his/her profit decreases. As the followers' decisions are influenced by the leader's strategy, the followers also do not tend to alter their decisions in the equilibrium point.

3. The assumptions and notations of the paper

3.1. The following notations are used in developing the proposed model:

$D_i(p_i)$	Demand rate for retailer i which is function of the retail price p_i	i	Index for retailers, i=1,..,n
D	Demand rate for the vendor $D = \sum_{i=1}^{n} D_i(p_i)$	S_{bi}	Fixed order cost for retailer i which is paid by the vendor ($/order)
Q	Vendor order quantity	S_v	Fixed order cost for vendor ($/set up)
q_i	Quantity dispatched to retailer i	ζ_i	Inventory cost paid to the vendor by retailer i ($/unit/time)
Q	Total number of items dispatched from the vendor to all retailers ($q = \sum_{i=1}^{n} q_i$)	π_v	Vendor's profit ($/time)
T	Vendor cycle time	π_i	Retailer i's profit ($/time)
T_i	Cycle time for retailer i	π_t	Total profit of the system ($/time)
T_R	Common cycle time for retailers	θ	Ratio of system's profit under applying a given contract to the systems' profit under centralized structure
Φ_i	Transportation cost for shipping the product from the vendor to retailer i ($/unit)	L_{bi} e_i k_i	Back order cost paid by the vendor to retailer i ($/unit/time) Price elasticity with respect to demand for retailer i Market scale for retailer i ($/unit)
Cm	purchasing cost of the product for the vendor ($/unit)	cm	purchasing cost of the product for the vendor ($/unit)
H_{bi}	holding cost paid by the vendor to manage the inventory of retailer i ($/unit/unit)	H_v	Holding cost at the vendor's site ($/unit/time)

The vendor's decision variables are as follows:

b_i	Fraction of backlogging per unit time for retailer i
q_i	Quantity dispatched to retailer i
N	Number of shipments received by a retailer
Cp	Wholesale price of the product

The retailer's decision variables are:

p_i	retail price charged by retailer i ($/unit)

In addition to these notations, λ denotes the percent of the retailers' revenue that must be paid to the vendor.

3.2. Problem statement

We consider a two-echelon supply chain that consists of one vendor and multiple retailers. The vendor orders an item from an outside supplier with unlimited stock, incurs an order cost S_v per lot and purchases the product from supplier by a fixed unit price cm. Vendor's warehouse capacity is also assumed unlimited. Demand for this product in the retailers' markets is assumed a decrease and convex function with respect to product price and can be described by Cobb-Douglas function as follows:

$$D_i(p_i)=k_i p_i^{-e_i} \quad ,i=1,...,n, \quad e_i>1 \qquad (2)$$

Cobb Douglas has been used frequently in the literature to show the relationship between price and demand (Yu Y. et al., (2009), Yu Y., (2009), Almehdawe E. and Mantin B. (2010)). In addition, retailers are independent from each other and do not compete in selling the product (for example, they are operating in the distinct markets). Vendor uses VMI strategy for controlling the inventory in the supply chain. Based on the VMI strategy, the vendor is responsible for controlling the inventory in the retailers' site and his own site. The relationship between vendor and retailers is leader-follower relationship, such that the vendor is the leader and the retailers are his followers. We assume that both of the manufacturer and different retailers are interested in establishing the long-term relationship. The VMI strategy reinforces such a relationship building once implemented. Firms are less likely to switch to a different party due to high switching costs. It is also assumed that the vendor replenishes retailers at the same time, that is $T_i=T_j=T_R$. This is a reasonable assumption in VMI environment because the vendor makes the decisions regarding the replenishment timing and amount.

Based on the WP contract, retailer i(i=1,…,n) pays cp+ ζ_i to the vendor per unit of product that is sold. The cp is the unit wholesale price of the product, which is a decision variable, and ζ_i is a parameter that is related to the inventory cost that retailer must pay to the vendor for controlling his inventory systems. Based on the revenue sharing contract, retailer i(i=1,..,n), must pay the wholesale price cp, for each unit of the product. He also should pay the related inventory cost ζ_i

and λ percent of his unit's retailer price to the vendor.
It is worthwhile to note that such VMI system have been proposed by Darvish and Odah (Darwish .M.A, Odah.M.O., 2010), but our view in modeling and analyzing of VMI system in this paper is quite different from the analyzed VMI system by Darvish and Odah.

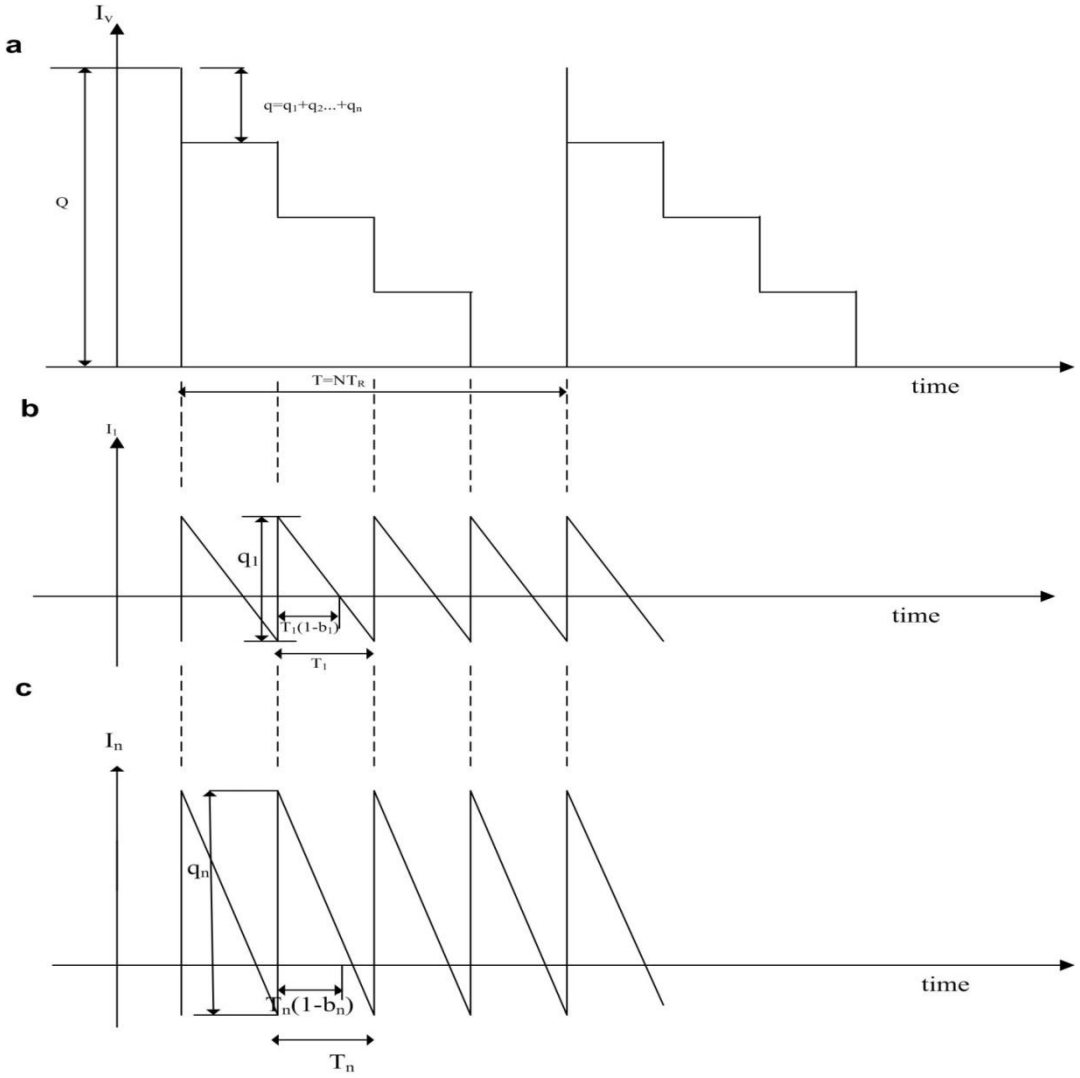

Figure 1. A supply chain consisting of one vendor and n retailer: Inventory levels against time for vendor, retailer 1 and retailer n

4. Net profits of the vendor and his retailers

In this section, based on the considered notations and assumptions in the previous section, the net profit functions of the vendor and his retailers are formulated. Fig1 depicts the inventory control chart in retailers' and vendor's sites. As mentioned earlier, such an inventory system has been analyzed from a different viewpoint by Darwish and Odah (Darwish .M.A, Odah.M.O., 2010).
As discussed in assumptions, vendor replenishes retailers at the same time. In other words, the replenishment cycles for each retailer are equal. Thus

$T_R=T_1=T_2=...=T_n$ (3)

Also it is clear that

$$T_i=\frac{q_i}{D_i};\quad i=1,2,...,n \tag{4}$$

Therefore, it is concluded that

$$T_R=\frac{q_1}{D_1}=\frac{q_2}{D_2}=...=\frac{q_n}{D_n} \tag{5}$$

On the other hand, for the vendor's replenishment cycle, the following is concluded

$$T=N.T_R=N.\frac{q_1}{D_1}=N.\frac{q_2}{D_2}=...=N.\frac{q_n}{D_n} \tag{6}$$

By expressing this preface, the net profits functions of each player are calculated in the supply chain.

The net profit of retailer i per unit time can be expressed by the following expression

$$\pi_i=D_i(p_i)\left[p_i(1-\lambda)-cp-\zeta_i\right] \tag{7}$$

In general, the vendor has two kinds of costs, direct and indirect costs. The direct costs include the purchasing costs of the product from the supplier and transportation costs to the retailers. Indirect cost is related to the retailers' and vendor's inventory systems. Therefore, direct costs of the vendor is obtained as follows,

$$TDC=\sum_{i=1}^{n}D_i(p_i)(cm+\Phi_i) \tag{8}$$

Now, consider the indirect costs faced by the vendor. The inventory holding cost, in each cycle T, incurred by the vendor, in his own warehouses, is as follows:

$$H_v\left[(N-1)q^2/D+(N-2)q^2/D+...+q^2/D\right]=H_v\frac{N(N-1)q_1^{\,2}D}{2D_1^{\,2}} \tag{9}$$

The total ordering costs in each cycle, T, is

$$S_v+N\sum_{i=1}^{n}S_{bi} \tag{10}$$

Also, the inventory holding costs, incurred by the vendor, in warehouse of retailer i in one cycle T_R, is as follows

$$\frac{D(p_i)T_R^{\,2}(1-b_i)^2}{2}H_{bi}+\frac{D(p_i)T_R^{\,2}b_i^{\,2}}{2}L_{bi} \tag{11}$$

Thus, the inventory holding cost in the site of retailer i, per unit time can be expressed by the following expression

$$THC_{bi}=\frac{q_1}{2D_1(p_1)}\left(D_i(p_i)(1-b_i)^2H_{bi}+D_i(p_i)b_i^{\,2}L_{bi}\right) \tag{12}$$

Thus, the total inventory holding cost in retailers' site per unit time is as follows:

$$THC=\sum_{i=1}^{n}THC_{bi} \tag{13}$$

Therefore, the total inventory systems' cost per unit time (sum of indirect cost) is obtained as follows:

$$TIC = \frac{q_1}{2D_1(p_1)}\left(\sum_{i=1}^{n}\left(D_i(p_i)(1-b_i)^2 H_{bi} + D_i(p_i)b_i{}^2 L_{bi}\right)\right) + H_v \frac{(N-1)q_1 D}{2D_1(p_1)} + \left(S_v + N\sum_{i=1}^{n} S_{bi}\right)\frac{D_1(p_1)}{Nq_1} \tag{14}$$

The vendor's total revenue is

$$\sum_{i=1}^{n} D_i(p_i)(cp + \zeta_i + p_i\lambda) \tag{15}$$

The vendor profit is the total revenue from all retailers minus the total costs faced by the vendor as described above. That is:

$$\pi_v = \sum_{i=1}^{n} D_i(p_i)(cp + \zeta_i + p_i\lambda) - TDC - TIC \tag{16}$$

5. Stackelberg game formulation of the system

In this section, we formulate the VMI supply chain when the vendor is the Stackelberg leader. Formulation of the Stackelberg game is developed based on the mentioned four steps in section 2. At the first step, according to the derived net profit function of each player in the previous section, the vendor and retailer's optimization problem are obtained. In the next step, by driving Karush-Kuhn-Tucker (KKT) conditions from retailer's optimization problem and adding these conditions to the vendor's optimization problem, the system's Stackelberg problem is formulated. The resulting Stackelberg game problem is a mixed integer non-linear problem (MINLP), where the optimal decisions of each part of the VMI supply chain and Stackelberg equilibrium can be determined by optimization methods.

The sequence of decisions in this Stackelberg game is as follows. In the first stage, the vendor as the Stackelberg leader determines the wholesale price of the product, the quantity to be sent to each retailer, the backorder fraction of each retailer, and the number of retailers' replenishments in each period. In the second stage, the profit maximizing retailers, as the followers, determines the retail prices in their corresponding markets. Therefore, the model formulation is as follows:

The vendor, who is the Stackelberg leader, solves the following optimization problem, denoted by *V*:

$$\begin{aligned} &Model\ (V) \\ &Max\ \pi_v = \sum_{i=1}^{n} D_i(p_i)(cp + \zeta_i + p_i\lambda) - TDC - TIC \\ &s.t.\ cp \geq cm \\ &\quad 0 \leq b_i \leq 1\ ,\ i = 1,...,n \\ &\quad N \geq 0,\ \text{integer} \end{aligned} \tag{17}$$

Where, the first constraint is a logical constraint. The second constraint is to set the limits for the fraction of backlogging (b_i) which cannot exceed 100% of the demand and the third constraint

defines N as an integer number.
Each retailer's profit maximization problem, denoted by F_i for retailer i, can be formulated as follows:

$$\begin{aligned} &Model\ (F_i) \\ &\max\ \pi_i = D_i(p_i)\left[p_i(1-\lambda) - cp - \zeta_i\right],\ i = 1,2,...,n \\ &Subject\ to\ \ p_i(1-\lambda) \geq cp + \zeta_i \end{aligned} \tag{18}$$

Where, the constraint guarantees at least positive profits for each retailer.
To find the optimal decisions, we first derive the Karush-Kuhn-Tucker (KKT) conditions for each of the retailers. For retailer i, these conditions are obtained as follows

$$\begin{aligned} &k_i p_i^{-e_i}\left[\frac{cpe_i}{p_i} + \frac{e_i\zeta_i}{p_i} + (1-\lambda)(1-e_i)\right] + r_i(1-\lambda) \leq 0 \perp p_i \geq 0 \\ &p_i(1-\lambda) - cp - \zeta_i \geq 0 \perp r_i \geq 0 \end{aligned} \tag{19}$$

Where, r_i is the dual variable for each retailer's constraint, and the $\perp$ symbol is used to show the orthogonality relationship between the followers' complementarity conditions.
Adding these KKT conditions to the vendor's optimization problem , Model *V*, while penalizing for the violation of the complementarity conditions in the objective function (M is the penalty) the resulting problem ,denoted by *L*, is a mixed integer nonlinear problem(MINLP).

$$\begin{aligned} &Model\ (L) \\ &Max\ \ \pi_v(N, cp, \lambda, q_1, b_1, ..., b_n, p_1, ..., p_n) = \\ &\quad \sum_{i=1}^{n} D_i(p_i)(cp + \zeta_i + p_i\lambda) - TDC - TIC \\ &-M\sum_{i=1}^{n} r_i\left[p_i(1-\lambda) - cp - \zeta_i\right] \\ &+M\sum_{i=1}^{n} p_i\left\{k_i p_i^{-e_i}\left[\frac{cpe_i}{p_i} + \frac{e_i\zeta_i}{p_i} + (1-\lambda)(1-e_i)\right] + r_i(1-\lambda)\right\} \\ &s.t.\ P_i(1-\lambda) \geq cp + \zeta_i,\ \ i = 1,...,n \\ &\quad 0 \leq b_i \leq 1\ \ ,i = 1,...,n \\ &\quad k_i p_i^{-e_i}\left[\frac{cpe_i}{p_i} + \frac{e_i\zeta_i}{p_i} + (1-\lambda)(1-e_i)\right] + r_i(1-\lambda) \leq 0\ \ ,i = 1,...,n \\ &\quad N \geq 0,\ \text{integer} \end{aligned} \tag{20}$$

Regarding model *L*, two points should be considered. First, by setting $\lambda = 1, cp = 0, \zeta_i = 0 (i = 1,...,n)$, the decentralized VMI supply chain structure becomes the same as the centralized supply chain structure and by solving this model, the system performance in the centralized supply chain is determined. Also in the case $\lambda = 0$, the RS contract changes to the WP contract and by solving this model; we can determine the system's performance under WP contract. Therefore, although this model formulates the supply chain under RS contract, in addition by solving this model, system performance in the centralized structure and under WP contract can be determined.

6. Numerical analysis of decentralized VMI supply chain

In this section, we conduct some numerical analysis to gain some insights regarding the outcomes of model L. We assess the sensitivity of results to changes in critical parameters of both parties (i.e., the vendor and retailers). To do this, three groups of parameters are taken into account, including those related to the vendor system (i.e., purchasing price and transportations cost), the retailers' markets (i.e., price elasticity and market scale) and the inventory control system (i.e., holding, backorder and ordering costs). All numerical analysis in this section is conducted for the case that its parameters are indicated in table 1. The parameters values of this case are randomly generated by the following suggestion from other researchers (Yu Y. et al., (2009), Yu Y. et al., (2009), Almehdawe E. and Mantin B. (2010)). The mathematical models were developed by Lingo 12 optimization package. Notably, in some reports, only the results for one retailer are illustrated for the sake of compression, while they are also correct for the other retailers. It is also worthwhile to note that we suppose $\lambda=0$ in model L in the analyses of this section because the impact of the λ is extensively analyzed in the next section.

Table 1. Parameters of the one vendor-three retailers' case

Retailers' data							Vendor's data		
k_i ($\times 10^4$)	e_i	ξ_i	Φ_i	H_{bi}	L_{bi}	S_{bi}	H_v	S_v	cm
390	1.2	5	6	6	500	100	1	150	50
160	1.5	8	5	8	300	120			
320	1.4	5	11	7	200	110			

6.1. Vendor's parameters

Here, we report the analytical results regarding the variations in parameters related to the vendor system (i.e., purchasing price and transportations cost).

***Vendor's purchasing price* (*cm*)**

Table 2 shows the effect of changes in vendor's purchasing price on the performance of VMI system partners. As it can be seen, by increasing *cm*, the profits of vendor, all retailers, and consequently the whole system will be decreased. Also, by increasing *cm*, the wholesale price increases and consequently retailers' prices increase also.

$$cm \uparrow \Rightarrow \begin{cases} cp,\ p_i & \uparrow \\ D_i(p_i), q_i, \pi_i, \pi_v, \pi_t & \downarrow \end{cases}$$

Table 2. The Influence of vendor's purchasing price on the VMI system performance

cm	*N*	q_1	*cp*	p_3	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)
Base	2	200	265	946	14.7431	17.5841	110.157
75	2	162	387	1371	12.7098	15.9454	100.56
100	2	139	509	1800	11.3988	14.8544	94.121
125	2	121	658	2319	10.3012	13.916	88.548

***Transportation cost* (Φ_i)**

Table 3 illustrates the impact of transportation cost on the performance of VMI system. The Transportation cost for each retailer is increased from the base value given in Table 1 by the values in the Table 3. As observed, by increasing the transportation cost, the wholesale and retailers' prices are increased. In contrast, the profits of vendor, retailers and the whole system will be decreased. Also, the value of transfer lot to retailer 1 is decreased because of an increment in retailers' prices and a decrement in retailers' demands. In brief, we can conclude that:

$$\Phi_i \uparrow \Rightarrow \begin{cases} cp,\ p_i \uparrow \\ D_i(p_i), q_i,\ \pi_i,\ \pi_v,\ \pi_t \downarrow \end{cases}$$

Table 3. The impact of transportation cost on VMI system performance

Φ	*N*	q_1	*cp*	p_3	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)
Base	2	200	265	946	14.7431	17.5841	110.157
5	2	192	286	1021	14.3051	17.239	108.129
10	2	182	314	1116	13.8029	16.8337	105.773

6.2. Retailers' markets parameters

In this subsection, we demonstrate the analytical results regarding the variations in retailers' markets parameters (i.e., price elasticity and market scale).

Price elasticity

Table 4 and figure 2 show the effect of change in e_1 (i.e., the price elasticity of retailer 1) on VMI system performance. As pointed, increments in e_1 result in a reduction in the prices and profits of

both vendor and retailer 1. Although raising e_1 also diminishes the prices of the other retailers, it can increase their profits instead. We note that the retailers' markets are independent; however, because parameter e_1 affects the vendor performance, the outcomes of the other retailers may naturally be influenced indirectly. According to equation 1, we cannot assess the direction of changes in the demands of retailer 1. Finally, the following observations may be deduced:

$$e_i \uparrow \Rightarrow \begin{cases} cp,\ p_i,\ \pi_i,\ \pi_v,\ \pi_t,\ p_j\ (j \neq i) \downarrow \\ D_j(p_j),\ q_j,\ \pi_j\ (j \neq i) \quad \uparrow \end{cases}$$

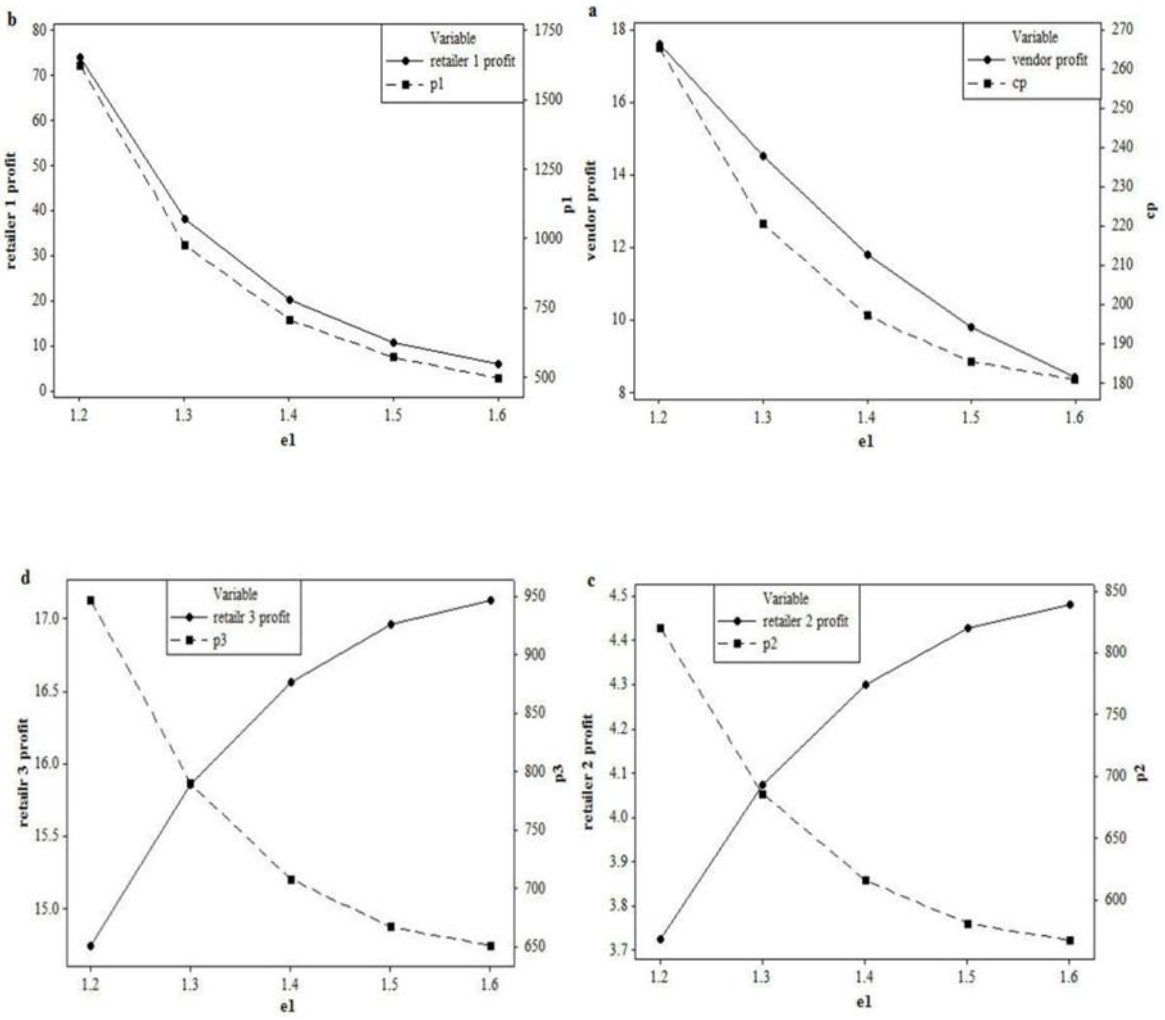

Figure 2. The impact of price elasticity on system performance (a) vendor (b) retailer 1 (c) retailer 2 (d) retailer 3

Table 4. Influence of price elasticity on VMI system performance

e_1	N	q_1	cp	p_1	p_2	p_3	π_1 (×10^4\$)	π_2 (×10^4\$)	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)
Base	2	200	265	1622	820	946	74.1054	3.72446	14.7431	17.5841	110.157
1.3	2	179	221	977	686	789	38.028	4.07361	15.8526	14.5196	72.474
1.4	2	144	197	708	616	708	20.1834	4.29908	16.5607	11.8121	52.855

Market scale

In table 5, we try to draw and analyze the impact of k (i.e., the market scale) on the system outcomes. Based on theorem 1, it can be seen that increasing k_2 leads to (1) the decrease in the wholesale and retail prices, (2) the increase in vendor's and retailer 2's profits. However, the changes in the profits of the other retailers are almost inconsiderable. This is because the demands of the other retailers are only influenced by the decrease in the wholesale and retail prices, while those of retailer 2 are also under effect of the market scale. Therefore, the changes in the demands of retailer 2 are definitely much more than those of the other retailers.
Therefore, in general, we can say that:

$$k_i \uparrow \Rightarrow \begin{cases} cp,\ p_i,\ p_j\ (j \neq i) \downarrow \\ D_i,\ q_i,\ \pi_i,\ \pi_v,\ \pi_t,\ D_j,\ q_j\ (j \neq i) \uparrow \\ \pi_j\ (j \neq i) \leftrightarrow \end{cases}$$

Generally, the retailers' market parameters and particularly the price elasticity have a significant effect on the system performance.

Table 5. The influence of market scale on the VMI system performance

k_2	N	q_1	cp	p_1	p_2	p_3	π_1 (×10^4\$)	π_2 (×10^4\$)	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)
base	2	200	265	1622	820	946	74.1054	3.72446	14.7431	17.5841	110.157
210	2	202	260	1593	806	929	74.3733	4.93217	14.8499	18.0466	112.202
260	2	205	256	1567	792	914	74.6247	6.15764	14.9504	18.5111	114.244
310	2	210	252	1542	780	899	74.8608	7.39936	15.0452	18.9773	116.283

6.3. Inventory control parameters

Our analysis indicates that parameters related to the inventory control system have less impact on the values of profits and prices in VMI system. However, these parameters could sometimes

influence the policy of inventory control system applied by the vendor. Table 6 gives the effect of inventory holding costs in the warehouse of retailers on the system performance. The value of H_{bi} is increased from the base value denoted in Table 1. We can see that when the holding costs in the retailers' sites are increased from the base value by 20 currency units, the number of replenishments increases from 2 to 3. Also, table 7 illustrates the effect of S_v (i.e., the ordering costs) on the system performance. Again, we observe that decreasing S_v from 150 to 100 leads to a reduction in N- i.e., from 2 to 1.

Table 6. The impact of holding cost in retailer side on the VMI system performance

H_{bi}	N	q_1	p_3	π_3 ($\times 10^4$\$)	π_v ($\times 10^4$\$)	π_t ($\times 10^4$\$)	cp
base	2	200	946	14.7431	17.5841	110.157	265
10	2	155	952	14.7035	17.521	109.942	267
20	3	123	957	14.6726	17.4719	109.775	268

Table 7. The effect of ordering cost in vendor side on the VMI system performance

S_v	N	q_1	p_3	π_3 ($\times 10^4$\$)	π_v ($\times 10^4$\$)	π_t ($\times 10^4$\$)	cp
base	2	200	946	14.7431	17.5841	110.157	265
100	1	221	945	14.749	17.5936	110.189	265

7. Analyzing the effect of the RS contract in the VMI supply chain performance

In this section we analyze the effect of RS contract on the VMI supply chain. Four different settings are considered for this contract:

RS contract I: both λ and cp are determined by the vendor and have been considered as the decision variables in Eq.20.

RS contract II: cp is determined by the vendor and has been considered as a decision variable, while λ is an agreed value between the vendor and retailers and has been considered as a parameter.

RS contract III: λ is a decision variable and is determined by the vendor but cp is a parameter that has an agreed value between the vendor and retailers.

RS contract IV: both cp and λ are the decision variables, where cp is determined by the vendor and the retailers determine λ.

In analyzing each of these RS contract settings, the changes are exerted on one of the two cases that their parameters are given in Table 8. The parameters values of these two cases are randomly generated by following suggestion from other researchers (Yu Y. et al., (2009), Yu Y. et al., (2009), Almehdawe E. and Mantin B. (2010)). Notably, in some reports, only the results for one retailer

are illustrated for the sake of compression, while they are correct also for the other retailers.

Table 8. Related Parameters of the two analyzed cases (the values of k are scaled on 10000)

case	k_i	e_i	ζ_i	φ_i	H_{bi}	L_{bi}	S_{bi}	H_v	S_v	cm
1	170	1.7	15	14	5	100	20	1	160	150
	380	1.7	7	5	8	200	20			
	260	1.6	12	15	8	300	30			
2	180	1.3	11	6	7	300	110	20	100	90
	240	1.5	11	6	5	100	150			
	280	1.2	15	11	6	500	110			

7.1. Revenue sharing contract I

Table 9 shows the performance of the system in the centralized setting (first row), under the WP contract (second row) and under the RS contract I (third row). In this contract, the changes are exerted on the case 2 of Table 8. The results have been shown in Table 9. It is concluded from Table 9 that the RS contract I has a considerable affect on improving the system's performance and makes it closer to the centralized structure. For example, the ratios of system's total profit under the RS contract I and WP contract to the centralized supply chain profits are 0.98 and 0.76, respectively. In addition, if we consider difference in prices in these three contracts, it is seen that, for example, retailer 1 sells its product in centralized contract, WP contract and RS contract I by 419 ,2064, and 557, respectively. Moreover, we can see from Table 9 that other decision variables in RSC I are very closer than WP contract to the corresponding value in centralized structure regarding the profits and prices.

Also, another result of profit analysis of RSC I is the drastic decrease in the retailers' profits in comparison with the WP contract. In general, the effect of this contract can be expressed as follows: the vendor sells product by a very low wholesale price (in this case it is zero) to the retailers, in return, he/she gets a high percent of the retailers' revenue (0.91 for this case). In fact, the vendor wants by decreasing cp and making it close to zero and on the other hand by increasing λ and making it close to one, makes the performance of the system close to the centralized structure as much as possible. This trend leads to the drastic decrease in retailers profit and the vendor gains much of the overall system profit. Furthermore, whenever ζ_i (i=1,..,n) equals zero, the vendor determines the optimal value of λ and cp equal to one and zero, respectively, that are boundary values for RSC I. Therefore, in this case, all the retailers' profits are zero and the performance of the system is the same as the centralized system's performance.

Table 9. case2: The optimal system performance under different supply chain structure (the value of profit is scaled on 10000 $)

Decision variable	N	q_1	cp	b_i	p_i	D_i	π_i	π_v	π_t	λ	θ
centralized	1	177 122 321	-	0.0228 0.04762 0.01186	419 290 610	700 486 1270	-	-	96.5201	1	-
WPC	1	61 30.5 136	456	0.0228 0.04762 0.01186	2046 1429 2882	88 44 197	14.0254 4.23257 47.4288	12.4038	78.12	0	0.76
RSC	1	159 104 217	0	0.0228 0.04762 0.01186	557 386 1052	485 316 662	1.77789 0.89722 4.96517	86.7467	94.187	0.91	0.98

7.2. RS contract II

In this sub-section, we analyze the effects of RS contract II, the situation that cp is determined by the vendor and λ as a parameter, is an agreed value between two sides of the contract (retailers and vendor). The value of λ may be a proposed value from one side of the contract or be an agreed value between two sides. As in many RS contracts, the retailer usually proposes λ (Cachon G.P., Lariviere M. A., 2005). In this contract, the changes are exerted on the case 1 of the Table 8. The system performance under centralized structure and WP contract, for this case, are illustrated in Table 10. Fig 3 shows the impact of changes in λ on the profits and prices. We can see that by increasing λ, the wholesale price and retail prices always decrease. Also retailers' profit decrease (in this case, the profit of retailer 3 is shown for example), while vendor's profit increases. In general, the effect of changing parameter λ can be expressed as:

$$\lambda \uparrow cp \downarrow p_i \downarrow D_i \uparrow q_i \uparrow \pi_i \downarrow \pi_v \uparrow \pi_t \uparrow$$

It is also concluded that the retailers receive product by lower wholesale price at the higher values of λ and the retailers' profits always decrease in spite of increasing λ. Also, at the higher value of λ, although vendor sells its product by lower wholesale price, by increasing the λ, the vendor's profit always increases. The vendor's stimulus to decrease cp in higher value of λ can be justified as follows: by decreasing the cp, retailers also decrease its retailer prices and therefore the overall demand of the system increase and consequently total profit of the system increases.

One weakness of RS contract II is the reduction of retailers' profits for each $\lambda > 0$. In fact, in determining the optimal value of λ, we encounter a paradox because from one side, the vendor's profit increases by increasing λ, while retailers' profit decreases by increasing λ. This causes RS contract II to be incapable of reaching to a win-win result for both sides. In Section 8, we present an initiative algorithm to eliminate this paradox.

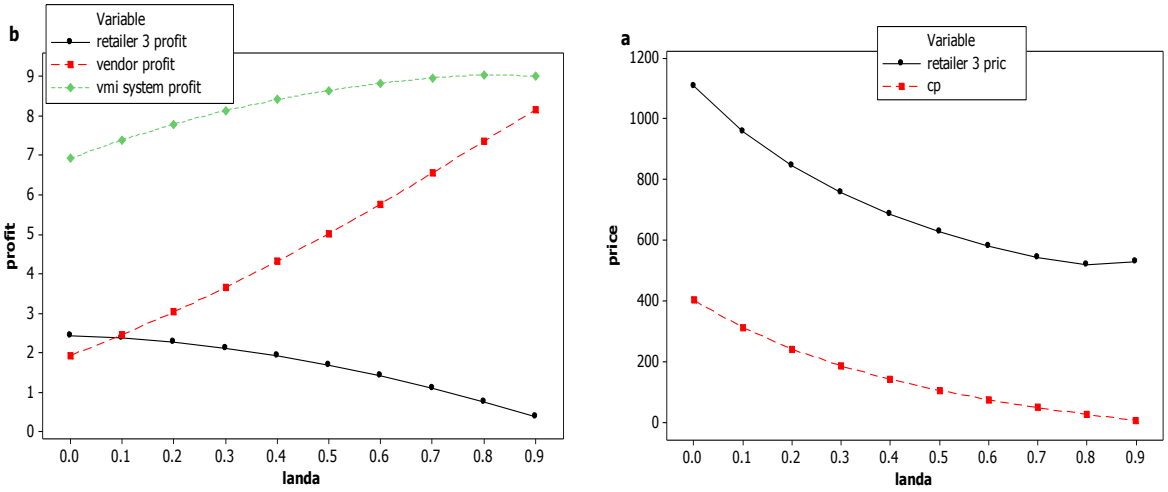

Figure 3. Case1: The influence of the λ on the (a) wholesale price and retailer 3' price (b) VMI systems retailer3's and vendor's profit for case 1(the values of profits are scaled on 10000$)

Table 10. case1: The optimal system performance under different supply chain structure (the values of profits are scaled on 10000$)

Decision variable	N	q_1	cp	b_i	p_i	D_i	π_i	π_v	π_t	λ	θ
centralized	4	15 51 49	-	0.04762 0.03846 0.02597	400 379 443	46 156 151	-	-	9.1546	1	-
WPC	4	7 16 19	402	0.04762 0.03846 0.02597	1013 994 1105	13 30 35	0.78676 1.78262 2.42548	1.94356	6.93841	0	0.76

7.3. RS contract III

In this sub-section, we analyze the case that λ is a decision variable and its value is determined by the vendor, while cp is a parameter. The value of cp may be a proposed value from one side of the contract or be an agreed value between two sides. If the optimal value of cp in WP contract is denoted by cp_0, it is obvious that for $cp>cp_0$ the retailers' profit in RS contract III becomes less than the corresponding value in WP contract (notice that $0 \le \lambda \le 1$). Therefore, the $cp>cp_0$ is impractical and no retailers are interested to participate in such a contract. Therefore, in this contract, we only focus on the case $cp<cp_0$. We recall that in many cases of the RS contract, the value of the wholesale price is less than the marginal cost of the product (Giannoccaro I. and Pontrandolfo P., 2004). Fig 4 illustrates the impacts of shifts in the parameter cp on the system performance. We can see that by increasing cp, the value of λ decreases and the retailers' profit

(in this case, the profit of retailer 3 is depicted) initially increases and then decreases slowly, while by increasing the cp, the vendor's profit always decreases. In general, the impact of this contract is obtained as follows

$$cp \uparrow \lambda \downarrow p_i \uparrow D_i \downarrow q_i \downarrow \pi_m \downarrow \pi_t \downarrow$$

Initially, by increasing the value of cp, the retailers attempt to increase their profits by increasing their retailer prices, but if the cp increases more than a special value, consequently the value of pi increases, therefore, the demand decreases such that increasing the value of price cannot prevent from decreasing in retailers' profit. In general, since increase in the cp leads to the decrease in overall demand of the system, therefore, by increasing the cp the vendor' and system's profit decreases too.

The significant result of this contract is as follows: selecting the value of cp that is less than the corresponding value in WP contract, cp0, does not necessarily increase the system's profit. For example, it is seen from Table 10 that the system profit under WP contract is approximately 700000 and wholesale price is 400, while in RS contract III, even for cp=200, the system profit is less than 700000. But for cp<100 (remark that cm=150) the total profit of the system is higher than WP contract (fig 3). This result is significant because in many RS contracts, retailers try to persuade the vendor to decrease his wholesale price less than its value in WP contract, and because of this decrease, they share the percentage of their profits with vendor (Cachon G.P., Lariviere M. A., 2005). Due to decreased wholesale price and consequently the retailer price, demand and profit of the system increases. Also, as depicted in fig 4, for cp<cp0, the profit of the retailer 3 is always less than its corresponding value in WP contract. It is also concluded from this contract that it is not expected that the vendor sets λ to zero where cp=cp0. For example you can see from the fig 4(a) that λ is more than 0.4 for cp=400. Thus, in general, setting cp<cp0 does not necessarily increase system profit, unless cp is chosen small enough (for example, less than marginal price of the product) but in this state the retailers' profit considerably decreases and the major portion of the supply chain's profit returns to the vendor.

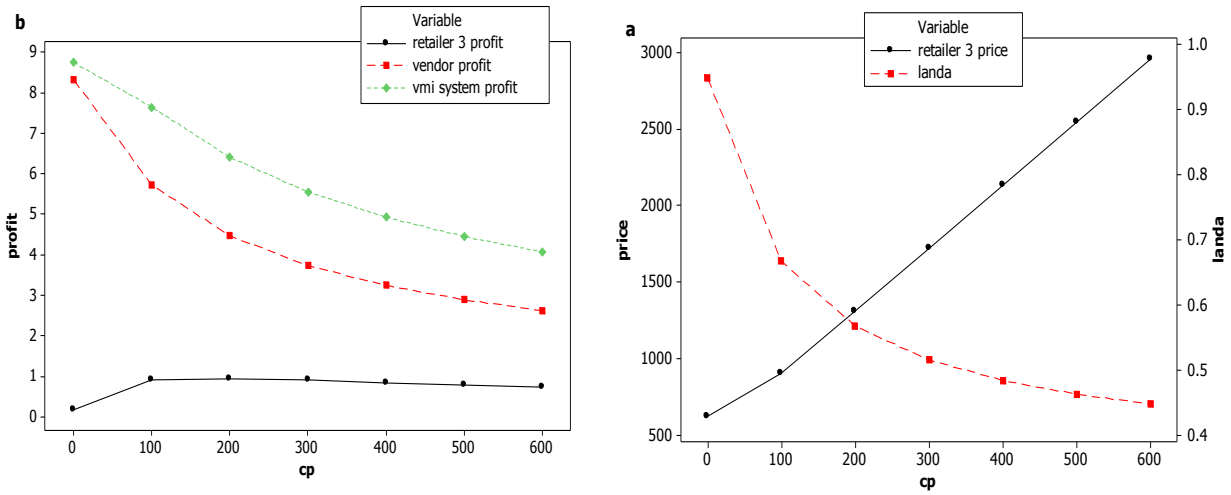

Figure 4. Case1: the influence of the cp on the (a) wholesale price and retailer3' price (b) VMI system's, retailer3's and vendor's profit (the values of profits are scaled on 10000$)

7.4. RS contract IV. If the retailers determine the value of λ, they always set its value equal to zero because the retailers' profit functions (Eq.7) are decreasing functions of λ. In this condition, RS contract is the same as the WP contract.

7.5. Comparing RS contract II and RS contract III

Although it seems that RS contract II is the reversed form of the RS contract III, there is a considerable difference between these two contracts. Suppose that in RSC II, the vendor and the retailers agree on one value of λ as λ_1 and in the next step, vendor set cp*(the optimal value of cp) equal to cp_1. Regarding these two contracts, following question arises.

In the RSC III, if both sides agree on the cp= cp_1, the vendor will set the λ equal to λ_1, As can be seen from fig.5 (a), the selected value of λ by the vendor is always higher than λ_1 in this case. This trend certainly causes a decrease in the retailers' profit in RS contract III, but it seems that it leads to an increase in vendor's profit. But by considering the fig.5(c), it is concluded that the vendor's profit is also less than its corresponding value in RSC II. In fact, though in RS contract III the vendor gets a more proportion of the retailers' revenue, his profit decreases compared to the RS contract II. Our Justification for this case is that if two sides agree on cp=cp_1 in RS contract III, then the vendor sets $\lambda > \lambda_1$, therefore the retailers increase their retailer prices for obtaining more profit and compensating the increase in λ. consequently, the overall demand of the system decreases, and this decrease leads to the loss for both the retailers and vendor. By decreasing the retailer's and vendor's profit, obviously the overall system profit decreases too (fig. 5(d)).

8. Determining λ in RS contract II and improving the contract performance

In this section, further to the assumptions given in section 3, it is assumed that all the retailers form an alliance and act together as a single player. According to the analysis presented about the RS contract II, it is concluded that decreasing the retailers' profit in the RS contract in comparison with the WP contract for any $\lambda > 0$, is one of the weaknesses of the RS contract II. In fact, we encounter a paradox in determining the optimal value of λ, because in one side, the vendor's profit increases by increasing the λ, such that vendor is interested in the values of λ close to 1, whereas retailers' profit decreases by increasing the λ, such that they always want the value of λ close to 0 as much as possible. This causes RS contract II to be incapable of reaching to the win-win result for both sides of the contract. Therefore, the main question in this contract is how to determine λ or how to agree on the specific value of λ. For determining the value of λ and eliminating mentioned paradox, we propose the following negotiation trend. By this change in RS contract II, this contract could reach a win-win result for both sides of the contract and therefore both sides have incentive to partnership in the contract.

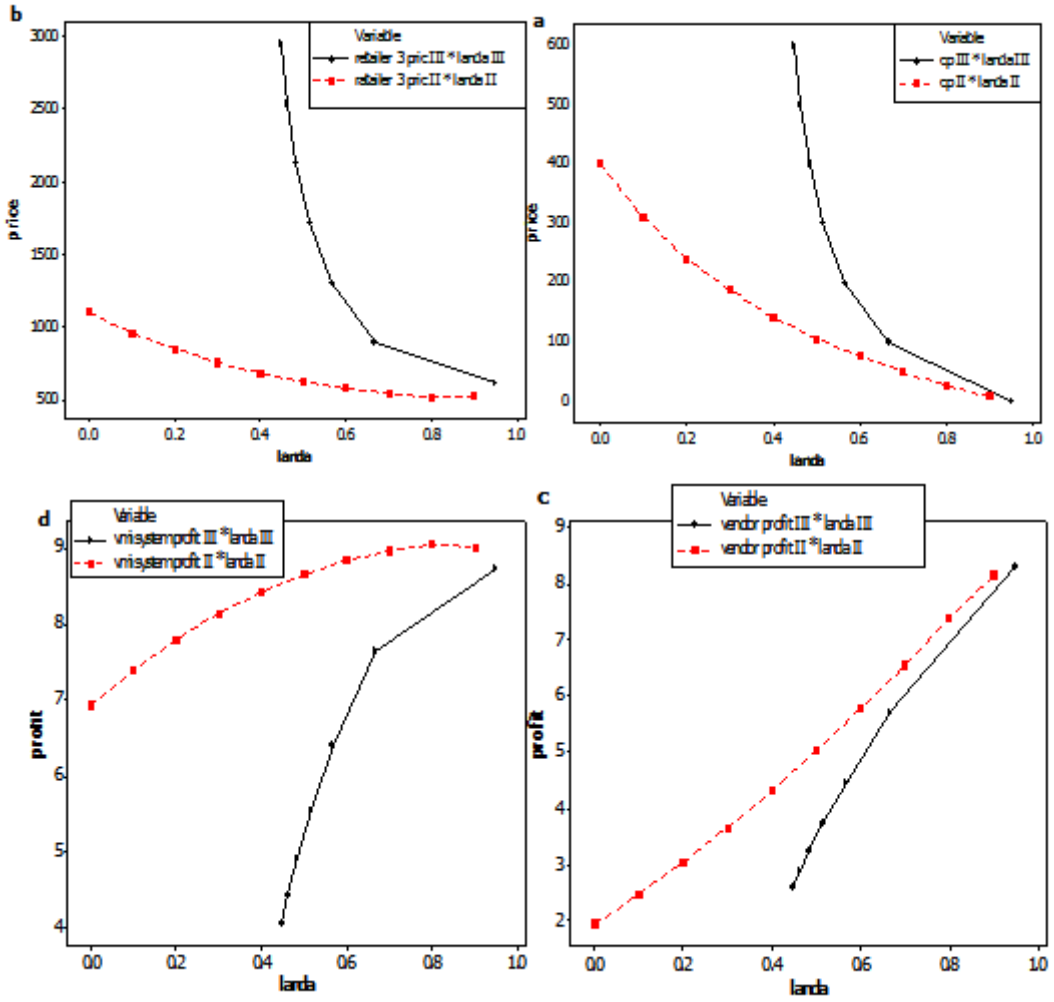

Figure 5. Case1: Comparing RSC II with RSC III (the values of profits are scaled on 10000 $)

Fig.6 shows, in general, the change in retailers' profit and vendor's profit with respect to change in λ for RSC II. It is assumed that, at this time, the two sides have made an agreement in the given value of λ such as λ_1. (This agreed λ can be 0). Assume that the vendor interests in increasing the λ from the current value, λ_1 to λ_2 ($\lambda_2 = \lambda_1 + \Delta\lambda$). This shift increases the vendor's profit, thus vendor always supports increasing λ, on the other hand this increase, decreases the retailers' profit and they will disagree with this increase. To convince the retailers to increase λ, the vendor is ready to pay to the retailers α percent of his added profit which is resulted from the increase in λ. Thus, the profits of the retailers and vendor in the new λ (λ_2) can be expressed as follows:

$$\pi_v^* = \pi_{v2} - \alpha(\pi_{v2} - \pi_{v1})$$
$$\pi_r^* = \pi_r + \alpha(\pi_{v2} - \pi_{v1}) \qquad (21)$$

In the new value of λ, if $\pi_r^* < \pi_{r1}$ the retailers refuse the new λ, because their profits decrease in

compared to the setting $\lambda = \lambda_1$. But, if $\pi_r^* \geq \pi_{r1}$, in this case, they agree to the new value of λ and we can continue to these steps by starting from the new value of λ. These steps are depicted in fig 7. By implementing this algorithm in RSC II, this contract can reach the win-win result for the participant in contract.

We give a simple example to clarify the mechanisms of the proposed algorithm. Assuming the value of α is 0.6. We analyze the case 1 in Table 8 for this example. Table 11 summarized the results of the proposed algorithm for this example. It is concluded from Table 11 that by increasing the λ from 0 to 0.1 and then to 0.2 the retailers' profit increases. But increasing the λ from 0.2 to 0.3 causes decrease in retailers' profit and therefore the optimal value of λ is 0.2.

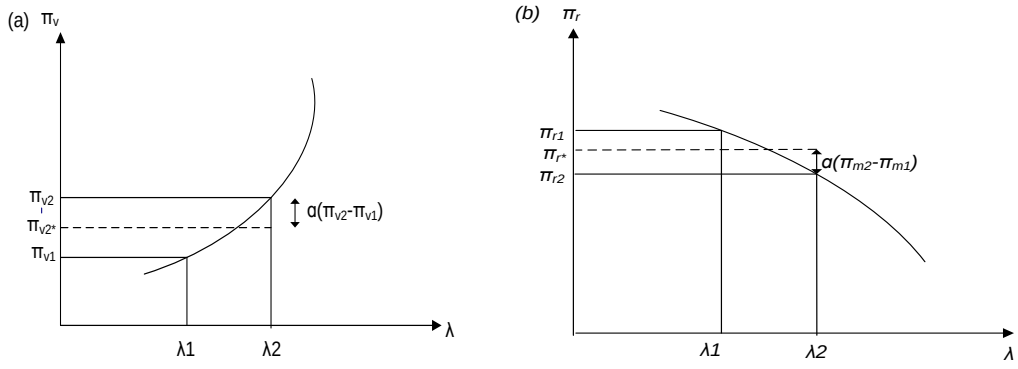

Figure 6. Change in (a) vendor's and (b) retailers' profit with respect to λ

Table 11. case1: Determining λ in RSC II based on the proposed procedure (the values of profits are scaled on 10000 $)

0	1.94356	4.99485			
0.1	2.46795	4.93871	0.52439	2.15332	5.25334
0.2	3.04048	4.76609	0.88716	2.50818	5.29839
0.3	3.6591	4.48616	1.15092	2.96855	5.17671
0.4	4.32205	4.10612	1.3535	3.50995	4.91822

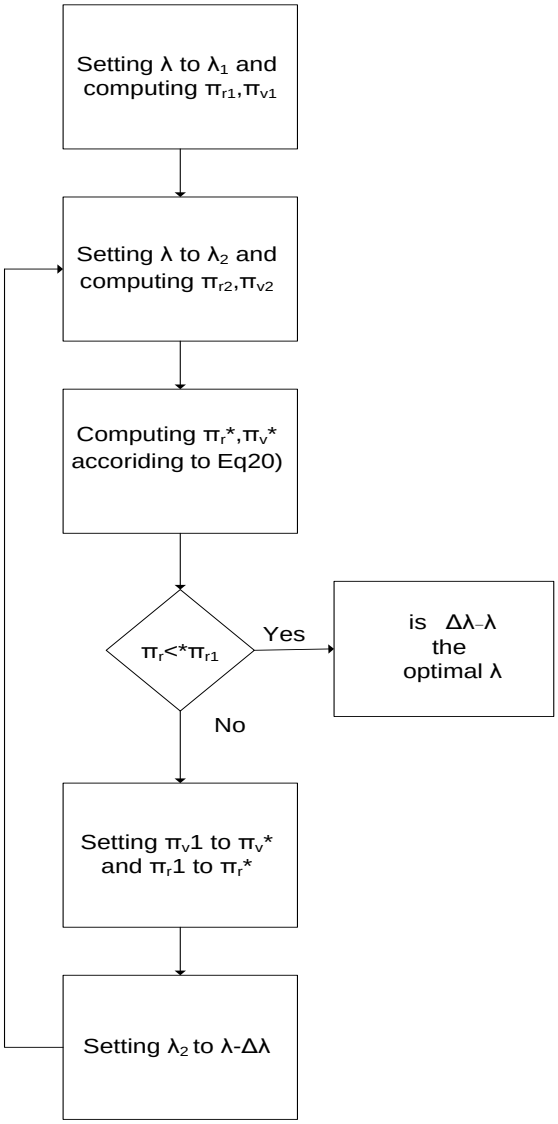

Figure7. The proposed algorithm for determining λ

9. Conclusions

In this paper, the performance of the RSC in VMI supply chain under the assumptions of Stackelberg game theory has been analyzed. The cost optimization model of the system is developed based on some assumptions about the VMI supply chain. Considering vendor as a leader in the game, formulating the Stackelberg game is performed based on the four steps. A general model is obtained which can explain system performance under RSC, WPC and centralized structure. Based on the fact that how the parameters of the RSC are determined, then four states for this contract are analyzed. Each state of RSC is analyzed by presenting convenient Tables and figures. Moreover, in different states, system performance under RSC compared with the system performance under WPC and centralized structure. Finally, a heuristic trend based on the negotiation is proposed for improving the effectiveness of revenue sharing contract. Also, sensitivity analysis is conducted with respect to three groups of the supply chain parameters: (i)

vendor's parameters (ii) retailers 'parameters and (iii) inventory related parameters. Results of our analysis indicate that some parameters such as price elasticity has a significant effect on the VMI system's performance, while some other parameters such as the inventory control parameters do not show such significant effects.

This article can be extended from the following directions:

1- Releasing some considered assumptions about the supply chain in section 3.2; such as vendor is a leader of the game or there is no competition between retailers.
2- Extracting closed form solution for the Stackelberg equilibrium based on the Eq.1 and then conducting numerical analysis.
3- Extending the proposed algorithm in section 7.

References

Almehdawe E. and Mantin B. (2010). Vendor Managed inventory with capacitated manufacturer and multiple retailer: Retailer versus manufacturer leadership. *International Journal of Production Economics*, 128, 292-302.

Braide S. Zcao Z. and Zeng (2013). Volume discount pricing strategy in the VMI supply chain with price sensitive demand. *Journal of the Operation Research Society*, 64, 833-847.

Bichescu.B.C. and Fry M.J. (2009). A numerical analysis of supply chain performance under split decision rights. *Omega The international journal of management science*, 37, 358-379.

Cachon G. P. (2003). Supply chain coordination with contracts. In: de Kok, A.G., Graves, S.C. (Eds.), Handbooks in Operations and Management Science, vol. 11. Elsevier, New York,.

Chaharsooghi K. and Heydari J. (2010). Supply chain coordination for the joint determination of order quantity and reorder point using credit option. *European Journal of Operational Research*, 204, 86-95.

Cachon G.P., Lariviere M. A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, 51, 30–44.

Chen J.M., Cheng H.L and Chien M.C. (2011). On channel coordination through revenue-sharing contracts with price and shelf-space dependent demand. *Applied Mathematical Modelling*, 35, 4886-4901.

Cachon P. G. , Netessine S. (2003). Game Theory in Supply Chain Analysis. *Supply Chain Analysis in the eBusiness.*

Darwish .M.A, Odah.M.O (2010). Vendor managed inventory model for single-vendor multi-retailer supply chains. *European Journal of Operational Research*, 204, 473-484.

Fiestras-Janeiro M.G., Garcia-Jurado I., Meca A., M.A. Mosquera.M.A. (2010). Cooperative game theory and inventory management. *European Journal of Operational Research*, 210, 459-466.

Giannoccaro I. and Pontrandolfo P. (2004). Supply chain coordination by revenue sharing contracts. *International Journal of ProductionResearch*, 89, 131-139.

Guan R. and Zhao X . (2010). On contract for VMI program with continuous review (r,Q) policy. *European Journal of Operational Research*, 207, 656-667.

Li S., Zhu Z. and Huang L. (2009). Supply chain coordination and decision making under consignment contract with revenue sharing. *International Journal of Production Economics*, 120, 88-99.

Mishra B. K. and Raghunathan S. (2004). vendor-managed inventory and brand competition. *Management Science*, 50, 445–457.

Nachiappan S P. and Jawahar N. (2007). A genetic algorithm for optimal operating parameters of VMI system in a two-echelon supply chain. *European Journal of Operational Research*, 182, 1433–1452.

Nagarajan M. and Sosic G. (2008). Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *European Journal of Operational Research*, 187, 719-745.

Pan K., Lai K.K., Stephen C.H.Leung and Xiao D. (2010) . Revenue-sharing versus wholesale price mechanisms under different channel power structures. *European Journal of Operational Research*, 203, 532-538.

Partha Sarathi G., Sarmath S.P and Jenamani.M. (2014). An integrated revenue sharing and quantity discounts contract for coordinating a supply chain dealing with short life-cycle products. *Applied Mathematical Modelling*, 38, 4120-4136.

Palsule-Desai. O.D (2013). Supply chain coordination using revenue-dependent revenue sharing contracts. *Omega*, 41, 780-796.

Qin Y., Tang H. and Guo C. (2007). Channel coordination and volume discounts with price-sensitive demand. *International Journal of Production Economics*, 105, 43-53.

Saha S.and Goyal.S.K (2015). Supply chain coordination contracts with inventory level and retail price dependent demand. *Int.J.Production Economics*, 161, 140-152.

Tyan H. and Wee H. (2003). Vendor managed inventory: a survey of the Taiwanes grocery industry. *Journal of Purchesung and Supply Management*, 9, 11-18.

Wang C.X. (2009). Random yield and uncertain demand in decentralised supply chains under the traditional and VMI arrangements. *International Journal of Production Research*, 47, 1955–1968.

Xu K. and Leung M T. (2009). Stocking policy in a two-party vendor managed channel with space restrictions. *International Journal of Production Economics*, 117, 271–285.

Yao Y. and Martin D. (2008). The inventory value of information sharing, continuous

replenishment, and vendor-managed inventory. *Transportation Research*, Part E 44, 361–378.

Yao Y., Dong Y. and Dresner M. (2007). Analyzing information-enabled stockout management under vendor-managed inventory. *Information Technology and Management,* 8, 133–145.

Yu Y., Huang G.Q. and Liang.L (2009). Stackelberg game-theoretic model for optimizing advertising,pricing and inventory policies in vendor managed inventory(VMI) production supply chains. *Computer & Industrial Engineering*, 57, 368-382.

Yu Y., Chu F. and Chen H. (2009). A stackelberg game and its improvment in a VMI system with a manufacturing vendor. *European Journal of Operational Research*, 192, 929-048.