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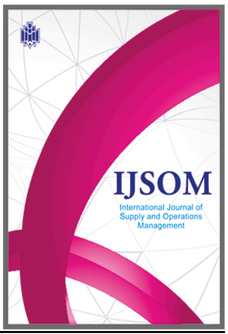

A Fuzzy Goal Programming Approach for Optimizing Non-normal Fuzzy Multiple Response Problems

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Abstract
In most manufacturing processes, each product may contain a variety of quality characteristics which are of the interest to be optimized simultaneously through determination of the optimum setting of controllable factors. Although, classic experimental design presents some solutions for this regard, in a fuzzy environment, and in cases where the response data follow non-normal distributions, the available methods do not apply any more. In this paper, a general framework is introduced in which NORTA inverse transformation technique and fuzzy goal programming are used to deal with non-normality distribution of the response data and the fuzziness of response targets respectively. Moreover, the presented framework uses a simulation approach to investigate the effectiveness of the determined setting of controllable factors obtained from statistical analysis, for optimization of sink mark index, deflection rate and volumetric shrinkage in plastic molding manufacturing processes. The accuracy of the proposed method is verified through a real case study.

Keywords: Non-normal multi- response optimization; NORTA Inverse Transformation Technique; fuzzy goal programming; injection molding simulation.

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1. Introduction

1.1. Design of Experiments

Design of experiments is a useful tool to improve the quality of different manufacturing processes. It discusses about determination of the controllable factors *X*s', which would result in the most desirable process outputs *Y*s', called as responses. The efficiency of design of experiments has been verified in many different manufacturing processes. For instance, Griffith et al. (2016) have used experimental design for optimizing the output quality of a 3D printing manufacturing process. In another research, Pawlak et al. (2017), designed some experiments for determining the optimum parameters of a laser melting process. Similarly, Chompu-Inwai et al. (2015), Sintov (2016), Guzman et al. (2016), Krohonen et al. (2016) and Giasin (2016) have used design of experiments in modern manufacturing researches.
One of the major fields of manufacturing in which the design of experiments has been widely used is plastic injection molding.

For instance, Oliaei Et al. (2016), Shiroudi-Heidari Et al. (2017), Chen et al. (2009) and Ozcelik et al. (2010) applied experimental design for optimization of plastic injection molding processes. It should be noted that most of the proposed methods only apply to problems in which the basic assumptions of classic design of experiments are realized.

1.2. Multiple Response Optimization

Although experimental design has been extensively applied by manufacturing researchers, there is a basic statistical assumption that limits applicability of design of experiments. This important limitation is that the classic methods only apply for single response problems where, the corresponding data follow normal distribution.

However, in the real world manufacturing processes these assumptions do not necessarily hold. For this regard, several methods are developed to optimize multi response problems. To see a comprehensive classification of multi-response optimization (MRO) methods see Amiri et al. (2012), and Bashiri et al. (2012).

In most of the methods as classified by Amiri et al. (2012), the multiple responses are transformed into a single overall response, and then the regression function of the overall response variables is fitted on the corresponding data. Hence, a function that illustrates the relationship between the overall responses with the controllable factors is achieved. Then this function is optimized to determine the optimum setting of controllable factors.

However, another approach for optimizing a multi response problem is to obtain the regression function of each response and then to use a multi objective method to optimize the functions (the regression functions of responses) to determine the optimum setting of controllable factors (Bashiri et al. 2013).

In addition to the statistical limitations of using design of experiments, in many real world applications, there are some sources of uncertainties in most manufacturing problems that the classic experimental design methods don't apply to them. These uncertainties always arise in cases where the process engineers cannot explain an exact target value for the response under study. Hence, it is more desired to state the target value as fuzzy numbers instead of crisp ones.

1.3. Fuzzy Logic

Bellman and Zadeh (1970), introduced the fuzzy set theory to overcome the problem of uncertainty in decision making problems. Tanaka (1974) proposed the concept of mathematical programming in fuzzy environment and Zimmerman (1978) reformulated linear programming for a single objective fuzzy problem.

In a multi objective optimization problem, an important issue should be considered and that is the preferences of the decision maker (DM) to prioritize the objectives according to the importance of each objective function. This importance can be modeled by transforming all objective functions into a scalar criterion by weights based on p-norm ($1 \leq p \leq \infty$) method. Also in many researches, the weighted additive model has been used for instance, see Chen and Tsai (2001).

In the real world examples the decision makers are always limited with imprecise knowledge about the process. Therefore, it is more appealing for them to state the importance of the objectives using linguistic terms such as very important, somewhat important or unimportant and so on. On the other hand, based on the lack of exact technical information, the desired value for the responses might be better to be mentioned by fuzzy numbers. In addition, since each objective might not exactly meet the corresponding target, it could be modeled using the fuzzy constraints such as $\tilde{\geq}, \tilde{\leq}$. In this case, the DM tends to maximize the degree of desirability, $\mu_{f_k(x)}$ for each $f_k(x), \forall k.$

1.4. Problem Statement and proposed solution strategy

As discussed above, rarely have been proposed researches in which experiments' design has been used for multiple response optimization problems with non-normally distributed data. Furthermore, due to imprecise knowledge in many manufacturing processes, fuzzy uncertainty exists in the model for which a reliable framework needs to be applied

The problem under study is so tangible in real world applications. To describe our problem using an example, consider a plastic injection molding in which the size and the number of surface defects of the part are the responses under study. While, the controllable factors of the process such as injection speed, temperature and pressure are considered. Since the number of surface defect follows non-normal distribution, a discrete distribution such as Poisson distribution, a transformation technic should be applied in the model before any statistical analysis so that all responses would follow normal distribution. This analysis, the design of experiments, is required to result two functions which show the relationships between each of the responses with the controllable factors. These functions show how controllable factors affect the response variables.

Clearly, the aim of the model is then to determine the setting of controllable factors in a way that the two responses are optimized simultaneously. In single response problems, conventional linear or non-linear mathematical methods such as the response surface methodology see (Khuri and Mukhopadhyay, 2010), could be applied.

However, in multiple response problems there would be multiple functions where each function represents how controllable factors define the corresponding response. Note that in multi-objective problems if each objective is optimized separately and regardless of other objectives, the obtained results can be misleading or even contradictory. For this reason, a multi objective optimization procedure such as goal programming (Charnes and Cooper 1977) should be conducted.

One important input of any multi-objective optimization algorithm is that the DM, should define an ideal target for the objectives, as well as assigning a weight to each objective to prioritize objectives. However, in the problem under study we assume the target values and the weight of the objectives are fuzzy numbers.

As a brief summary of the innovations of this article, the proposed framework facilitates a systematic procedure for applying design of experiments in multiple response optimization with non-normally distributed data. Moreover, in the cases where the ideal value of each response is determined as a fuzzy number and the priority of the response variables are stated using linguistic terms, this framework works well. In addition, this article proposes a simulation based analysis to examine the performance of a predicted optimum treatment on the important measurements in plastic manufacturing processes.

This paper is organized as follows: In the second section the, proposed methodology is explained. In the third section, a real case study is presented and the method is implemented on the real case step by step where the efficiency of the proposed method is verified in a real world case study. Besides, using a simulation approach the results of proposed method are examined in the fourth section. Finally, the conclusions are discussed in the fifth section.

2. Proposed methodology

The proposed method consists of five steps. The first step is designing and conducting the experimentation, and the second step is transformation of non-normal responses into normally distributed data. In the third step, the regression function for each of the responses is fitted on the corresponding data. Considering the fuzzy goal programming concepts, the fuzzy goal programming model of the problem is built and optimized in the fifth section to determine the optimum setting of controllable factors. The steps of the proposed methodology are illustrated in the Figure 1.

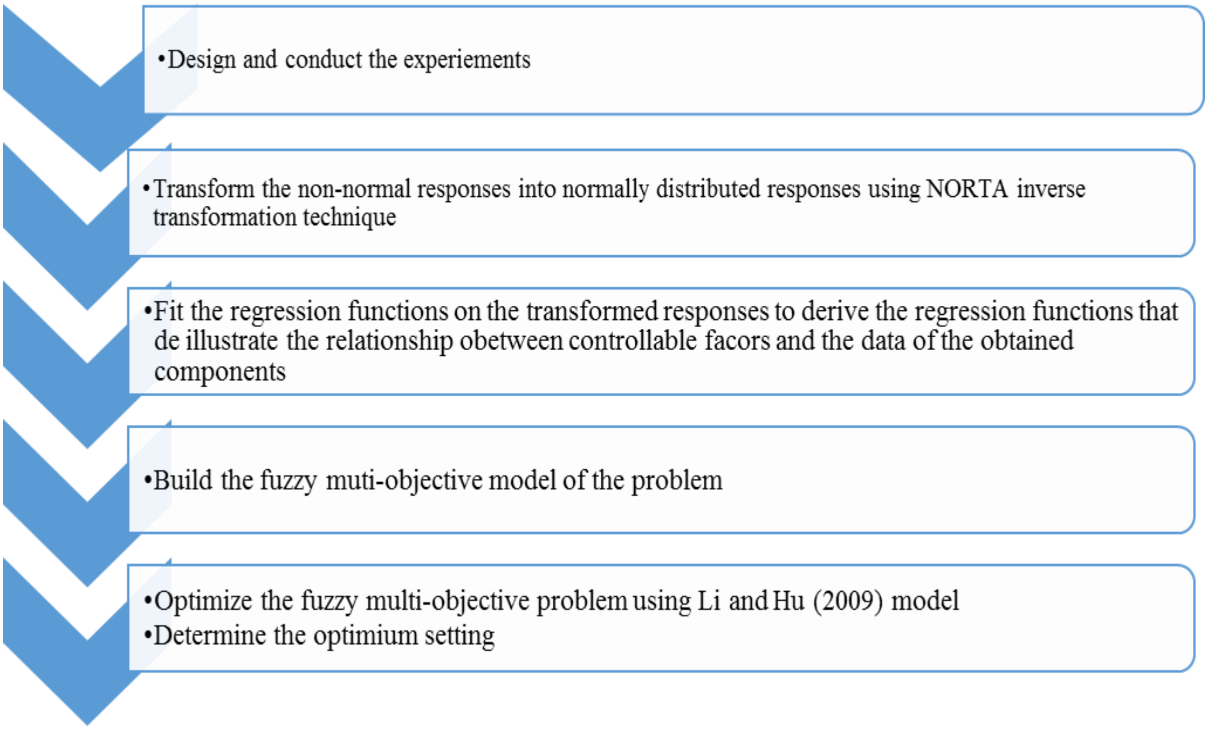

Figure 1. The proposed methodology

2.1 Experimental design

In the first step the experimentation is designed, planned and conducted. For this regard, according to controllable factors and their corresponding levels, a set of treatments is designed. There are different designs for experimentation, such as Taguchi orthogonal arrays or factorial designs or fractional factorial designs and etc. Choosing a design method depends on several parameters like cost, time and technical constraints. Although statistically it is recommended to experiment the full combination of controllable factors, but regarding to the constraints of time, cost and other limitations, fractional designs like Taguchi method are preferred in real world applications. After choosing the appropriate design, the experimentation phase is conducted and the response data are collected. In this paper, we use Taguchi experimental design for its simplicity and applicability.

2.2 Transformation of non-normal responses

In this step, non-normal responses should be transformed into normal ones. Since all the developed methods of design of experiments are based on the normality of responses, applying these methods for non-normal responses leads to misleading results. Hence, the responses should be transformed into normally distributed responses before the analysis to obtain more reliable conclusions.

In statistical process control, some transformations have been used. For instance, Box and Cox (1964) and Johnson (1949) are the basic methods proposed for transforming non-normal data to normal ones. Quesenberry (1997) proposed the Q-transformation method, and Xie et al. (2000) introduced double square root transformation. For this purpose the more recent method which is NORTA inverse transformation proposed by Niaki and Abbasi (2009), was used for transformation of a multi-attribute data to a multivariate normal distribution. NORTA inverse technique transforms non-normal multivariate data to approximate multivariate normal distributed data.

In this paper, the NORTA inverse transformation method is used for transformation of non-normal data to multivariate normal responses. Although there are several other transformation techniques in the literature, but Niaki and Abbasi (2009) showed that when the data follow discrete distributions, NORTA inverse transformation gives better results in comparison to other techniques. For transforming the non-normal vector of *n* correlated responses of $y = (y_1, y_2, \ldots, y_n)$ to the multivariate standard normal distributed vector of

$y' = (y'_1, y'_2, \ldots, y'_n)$, Equation (1) is used.

$$y' = (y'_1, y'_2, \ldots, y'_n) = ((\Phi^{-}\left(F_{Y_1}(y_1)\right), (\Phi^{-}\left(F_{Y_2}(y_2)\right), \ldots, (\Phi^{-}\left(F_{Y_n}(y_n)\right)). \quad (1)$$

In this formula, Φ^{-1} represents the inverse cumulative density function (CDF) of standard normal distribution. F_{y_j} is the cdf of y_j .Therefore, each treatment is considered as a certain process and accordingly the required parameters of the data distribution are estimated using the data collected in each treatment.

2.3. Fuzzy multi-objective modeling of the problem

After transforming the initial responses into normally distributed ones in the previous step, it is possible to perform a regression analysis for each of these responses separately. The result of this step is estimating the regression function of each response in terms of controllable factors, *Xs*', as shown in equation (2) as follows:

$$\hat{y}_1 = \hat{f}(x_1, x_2, \ldots, x_m). \quad (2)$$

One of the main results of the experimental design is that this analysis can formulate the response variables in terms of controllable factors as shown in Equation (2). The achieved equation is important from the point of the view that it gives the analyzer an insight of how controllable factors affect the response variable.

Of course, to determine the optimum setting of controllable factors that gives the best value of the response variable, a mathematical optimization needs to be performed. In single response problems, the methods such as response surface methodology (RSM) (Khuri and Mukhopadhyay, 2010) are applied. However, in multiple response problems there are more objectives to be optimized simultaneously.

Although many multi objective optimization methods have been developed (Marler and Arora, 2004) but in all these methods the DMs are supposed to define goals, the target values for objectives, as well as to prioritize the objectives. However, as mentioned before, in many applications, the targets are assumed to be fuzzy numbers, so an objective desirability needs be computed according to the target instead of a crisp number.

In cases where the responses have fuzzy targets, due to its type, the corresponding defuzzification method should be applied. This defuzzification is mostly a sort of mathematical modeling to assign a membership value to each value of a response.

Considering the type of the response that can be smaller the better (STB) or larger the better (LTB) or nominal the best (NTB) different membership functions are used. For STB objectives, the membership function is given as equation (3). Figure 2 illustrates this membership function.

$$\mu_{f_k(x)} = \begin{cases} 1 & f_k(x) \le f_k^* \\ 1 - \dfrac{f_k(x) - f_k^*}{f_k^{\max} - f_k^*} & f_k^* \le f_k(x) \le f_k^{\max} \\ 0 & f_k(x) \ge f_k^{\max} \end{cases} \tag{3}$$

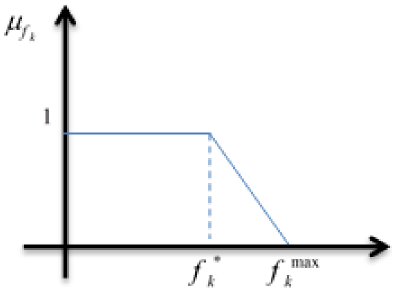

Figure 2. Membership function for smaller the better (STB) objective.

where f_k^* is the most desirable value for the *k*th objective. $f_k^{\max}$ represents the maximum value for the *k*th objective that can be accepted .The formulas for computing the membership function for LTB objectives and NTB objectives are shown in equations (4) and (5) respectively. The corresponding membership functions are also depicted in Figures 3 and 4. In this formula $f_k^{\min}$ is the minimum value for the *k*th objective that might be accepted.

$$\mu_{f_k(x)} = \begin{cases} 1 & f_k(x) \ge f_k^* \\ 1 - \dfrac{f_k(x) - f_k^{\min}}{f_k^* - f_k^{\min}} & f_k^{\min} \le f_k(x) \le f_k^* \\ 0 & f_k(x) \ge f_k^{\max} \end{cases} \tag{4}$$

$$\mu_{f_k(x)} = \begin{cases} 0 & f_k(x) \le f_k^{\min} \\ 1 - \dfrac{f_k(x) - f_k^*}{f_k^{\max} - f_k^*} & f_k^* \le f_k(x) \le f_k^{\max}, \\ 1 & f_k(x) = f_k^*, \\ 1 - \dfrac{f_k(x) - f_k^{\min}}{f_k^* - f_k^{\min}} & f_k^{\min} \le f_k(x) \le f_k^*, \\ 0 & f_k(x) \ge f_k^{\max}, \end{cases} \tag{5}$$

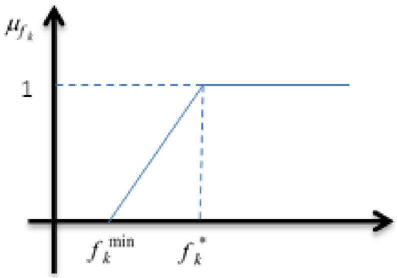

Figure 3. Membership functions for larger the better (LTB) objectives.

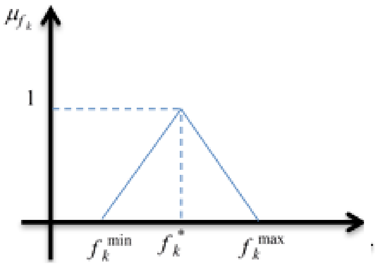

Figure 4. Membership functions for nominal the best (NTB) objectives.

After determination of fuzzy desired values of the responses, a multi objective problem is available to be optimized. In this research, the model proposed by Li and Hu (2009) is used according to the fuzzy desired values of objectives and their fuzzy priorities. The effectiveness of this method is evaluated by Rabbani et al (2016).
Consider $f_k(x)$ is STB type objective for which we are interested to find a solution that results in $f_k(x) \le f_k^*$, however, we approve other solutions which result in $f_k^* < f_k(x) \le f_k^{\max}$ with less desirability. The illustration of such objective was shown in Figure 1. The corresponding constraint for such an objective is shown as equation (8) as follows:

$$f_k(x) \tilde{\lesssim} f_k^*. \tag{6}$$

When $f_k(x) > f_k^*$, a surplus variable shown as p_k exits which causes the distance of the $f_k(x)$ from f_k^*, as shown in equation (7). This surplus variable should be minimized so that $\mu_{f_k(x)}$ is maximized. Using the mentioned concept, the defuzzification method of fuzzy STB objectives is shown as equation (8).

$$f_k(x) - p_k = f_k^* \tag{7}$$

$$\mu_{f_k(x)} = 1 - \frac{p_k}{f_k^{\max} - f_k^*} \tag{8}$$

If $f_k(x)$ is LTB type objective with the most desired value of f_k^* and the lowest limit value of $f_k^{\min}$, as shown in Figure 2, the corresponding fuzzy constraint, slackness equation and its defuzzification equation are shown in (9), (10) and (11) respectively.

$$f_k(x) \tilde{\gtrsim} f_k^*, \tag{9}$$

$$f_k(x) + n_k = f_k^*, \tag{10}$$

$$\mu_{f_k(x)} = 1 - \frac{n_k}{f_k^* - f_k^{\min}} \tag{11}$$

Similarly for NTB type objectives the corresponding equations are shown in (10), (11) and (12) respectively.

$$f_k(x) \cong f_k^*, \tag{12}$$

$$f_k(x) + n_k - p_k = f_k^*, \tag{13}$$

$$\mu_{f_k(x)} = 1 - \frac{n_k}{f_k^* - f_k^{\min}} - \frac{p_k}{f_k^{\max} - f_k^*} \tag{14}$$

In the model proposed by Li and Hu (2009), in addition to that each function has fuzzy targets, the objective functions have priorities in comparison to each other. In other words, some objectives are more important than the others. However, the priorities of objectives are also stated in terms of linguistic terms such as somewhat important and important. To consider this issue of the problem fuzziness, importance of objectives are incorporated in the model using a variable notated as γ. This variable defines this difference between the objectives with neighborhood linguistic importance weight. Neighborhood importance means that two objectives are just in one level of priority different, such as the linguistic terms of very important and somewhat important.

For instance, consider $f_k(x)$ is a very important objective with the satisfaction degree of $\mu_{f_k(x)}$, and $f_j(x)$ is a somewhat important objective with the satisfaction degree of $\mu_{f_j(x)}$. Since the linguistic terms of very important and somewhat important are neighbor linguistic terms, to show that optimization of the first objective is more important than the second one, the constraint shown as equation (15) can be added in the model.

$$\begin{aligned} &\mu_{f_j(x)} - \mu_{f_k(x)} \le \gamma, \\ &\gamma \le 0. \end{aligned} \tag{15}$$

Note that considering the preservation of the priority differences of objectives in the model, is in contrast with the goal of maximization of overall satisfaction degrees. For this reason, in the objective function of the model proposed by Li and Hu (2009), a term of $-\omega.\gamma$ is added, which describes this contrast. ω is a constant variable defined by the user. In other words, a tradeoff between the maximization of the overall satisfaction degrees and the preservation of the priority differences of the objectives should be considered. This tradeoff is modeled in the objective function of the model as shown in equation (16).

$$\max z = \left(\left(\frac{1}{p}\sum_{k=1}^{p}\mu_{f_k(x)}\right) - \omega\gamma\right) \tag{16}$$

All the elements of the model proposed by Li and Hu (2009), were explained in equations (4) to (16). The whole model is shown in equation (19). In this model the STB objectives are represented using the notation of S=1,2,…,s., the LTB objectives are represented using L=s+1,…,l index and the NTB objectives are shown using the N index, N= l+1,…, n. This partitioning is just performed for more explanation of the model.

$$Max \quad {}^{1}\!/_{p}\,(\sum_{S=1}^{s}\mu_{f^*_s} + \sum_{L=s+1}^{l}\mu_{f^*_i} + \sum_{N=1+1}^{p}\mu_{f^*_n}) - \omega.\gamma \tag{17}$$

s.t.

$$f_k(x) + n_k - p_k = f_k^*, k = 1,2,..,p \tag{18}$$

$$1 - \frac{p_s}{(f_s^{\max} - f_s^*)} \geq \mu^*_{f_s}, \forall S = 1,2,...,s \tag{19}$$

$$1 - \frac{n_l}{(f_l^* - f_l^{\min})} \geq \mu^*_{f_l}, \forall l = s+1,...,l \tag{20}$$

$$1 - (\frac{n_n}{(f_n^* - f_n^{\min})} + \frac{p_n}{(f_n^{\max} - f_n^*)}) \geq \mu^*_{f_n}, \forall N = l+1,...,p \tag{21}$$

$$\mu^*_{f_q} - \mu^*_{f_j} \leq \gamma, \forall q, j, \tag{22}$$

$$n_l \leq f_l^* - f_l^{\min}, \forall l = s+1,...,l. \tag{23}$$

$$p_s \leq f_s^{\max} - f_s^*, \forall s = 1,...,s. \tag{24}$$

$$n_i, p_s, \mu^*_{f_k} \geq 0, n_l.p_s = 0; \tag{25}$$

$$X \in G, \tag{26}$$

$$s + l + n = p, \tag{27}$$

$$n_k.p_k = 0, \forall k = 1,2,...,p. \tag{28}$$

In the model, $X \in G$ refers to technical constraints of controllable factors. By solving this model using different solvers like LINGO software, the most compromising setting for controllable factors is achieved. In the next section, the presented model is applied in a real case study.

3. Case study

Maadiran plastic factory produces plastic parts using injection molding processes. For producing a front cabinet of a monitor, two quality characteristics are defined as critical responses. The first response is the size of lower side and the other one is the number of sink marks on the part surface.

Sink mark is an injection defect which is caused by different shrinkage ratio of part's different zones. To reduce the number of sink marks, the shrinkage of the part should be controlled, and due to that the shrinkage can affect the size of the part, the shrinkage is the mutual factor affecting the number of sink marks as well as the size of the part.

According to the environmental imprecise factors, such as material nuisances, weather conditions and etc., it is more reasonable to assume that the target size of the part is considered as a fuzzy number as well as the target number of sink. The size is normally distributed by the mean and standard deviation of 497.4 and 0.37 *mm* respectively. Its target is represented as the fuzzy number of [497.5 *mm*, 497.8 *mm*,497.8 *mm*] in which the size of 497.8 has the membership degree equal to one and the membership decreases linearly until zero for the size of 497.5. It is clear that the size can be considered as an LTB variable. The sink mark follows Poisson distribution and its target is the fuzzy number of [0, 0, 1.5] and can be considered as an STB response.

3.1. Experimentation

The controllable factors and their levels are shown in Table 1 to determine which factors have significant effect on the process output.

Table 1. The controllable factors and corresponding levels

Factors	Levels
Barrel Temperature (BT)	230,235(°C)
Holding Time (TRH)	3,4,5Seconds
Holding Pressure (PH)	70%,80%,90% (maximum machine pressure)
Injection Speed (VH)	50%,60%,70% (maximum machine injection speed)
Injection Pressure (PP)	45%,50% ,55% (maximum machine pressure)
Input Parameters:	
Injection Machine: LG, 550 Ton.	Material: ABS, HF 380.

Based on the selected factors and corresponding levels, an $L18$ Taguchi design is chosen for experimentation. Note that the controllable factors impact the plastic part properties which were explained in section 3, and the properties impact the responses defined in the case under studied. Since these properties cannot be measured in act, the size and the number of sink marks are just considered in the experimentation.

The experimentation results are reported in Table 2. In the conducted experiment each treatment is replicated 10 times.

Table 2. Experimental results of the mentioned case study

Trt.	Factors Levels			Response	*rep*=1	*rep*=2	.	*rep*=9	*rep*=10
1	BT=230	TRH=3	PH=70	Size	498.04	498.05		498.36	498.07
	VH=50	PP=45		Defect	2	0		3	5
.	.	.	.	.					
18	BT=235	TRH=5	PH=90	Size	497.8	498.06		498.16	498.37
	VH=60	PP=45		Defect	0	1		0	3

3.2. Transformation of non-normal responses

In this step, the second response which is the number of sink marks on each part should be transformed into a standard normal distributed response. Hence, using equation (1) the equivalent point of standard normal distribution is computed for each of Poisson data. Note that, for computing the *cdf* of the data, the mean value of all replicated data in each treatment is considered as an estimate of the λ parameter of Poisson distribution ($\hat{\lambda}$). Some results of mentioned procedure for the case study are given in Table 3.

Table 3. Transformation results of the mentioned case study

Trt.	*rep*=1	.	*rep*=10	$\hat{\lambda}$	CDF *rep*=1	.	CDF *rep*=10	normal *rep*=1		normal *rep*=10
1	2		5	1.9	0.703	.	0.987	0.535		2.22
.						.				
18	1		3	1.4	0.592	.	0.946	0.232		1.61

3.3. Mathematical modeling of the problem

In this step the regression function for each of responses is determined separately. Hence, the responses are regressed on the controllable factors. The fuzzy desired values for these objectives are available from the previous step. The estimated regression functions are given in Table 5.

Table 5. The regression functions for the responses in the case study

Row	Regression function
1	$\hat{y}_1 = 353 - 0.0086BT - 0.047TRH + 0.00223PH + 0.01441VH - 0.0155PP$
2	$\hat{y}_2 = 357 - 0.0283BT - 0.0747TRH - 0.0006PH + 0.01971VH + 0.0143PP$

At this stage of the proposed method, the regression functions which illustrate the relationship between controllable factors and the responses are obtained. These estimated functions can be analyzed as a multi objective problem with fuzzy target values. From this step forward we are dealing with a fuzzy multi-objective problem derived from an experimentation phase. In the mentioned case study, two objective functions should be optimized for which the targets are fuzzy numbers. Besides, the importance of each objective is determined using linguistic terms where, the priorities of the first and the second objectives are stated as very important and somewhat important respectively.

The fuzzy goal programming model for this case can be formulated as follows. The ω parameter is determined equal to 0.1 by the DM.

Max=$(((\mu_{f^*_1} + \mu_{f^*_2}) / 2 - 0.1\gamma$

$$353 - 0.0086BT - 0.047TRH + 0.00223PH + 0.0144VH + 0.0155PP + n_1 - p_1 = 351.88$$

$$357 - 0.0283BT - 0.0747TRH - 0.0006PH + 0.0197VH + 0.0143PP + n_2 - p_2 = 352.62$$

$$1 - n_2 / 0.83 >= \mu_{f_2^*},$$

$$\mu_{f_2^*} - \mu_{f_1^*} <= \mu_{f_2^*} - \mu_{f_1^*} \leq \gamma,$$

$n_1 p_1 = 0$,
$n_2 p_2 = 0$,
$BT >= 200$,
$TRH >= 3$,
$PP >= 20$,
$TRH <= 9$,
$PH >= 50$,

By solving this model the optimum value for the controllable factors are obtained as $BT = 200$ °C, *TRH =3s, PH=50%, VH=63.5%, PP=20%* Respectively. To assure the accuracy of the determined setting, a verification experiment is conducted in 30 runs and the mean value of the size of the lower side is equal to 497.78 and the number of sink marks is equal to 0.233 on each part respectively.

These values represent an approximate achievement degree equal to 0.8, which is a permissible result in comparison with the predicted average achievement degree of the responses, $\mu_{f_1^*}, \mu_{f_2^*} = 1$.

These results verify the applicability of the proposed method.

In plastic injection molding processes, most researchers tend to evaluate the effectiveness of a proposed setting of controllable factors on a few crucial properties of the plastic part in simulation analyses as well. These properties are deflection, sink mark and the volumetric shrinkage of the part which are not easily measurable in real experimentations, so a simulation software is used to facilitate the analysis of the mentioned properties.

4. Simulation Analysis

In this section the modeling of the simulation is discussed. The modeling of the problem includes modeling the runner system and the cooling system explained as follows:

The runner system is designed and modeled as presented in Figure 4. Four gates are designed to distribute the melt in the cavity to keep the pressure drop and flow in the cavity as steady as possible. The distances between gates are also designated to provide uniform filling in sub section of the part. The cooling circuit is also modeled as presented in Figure 5.

The presented cooling system is designed in two directions with two water inlets (x and y) which results in higher heat removal efficiency as well as even cooling compared with the system only in one direction (see Figure 6).

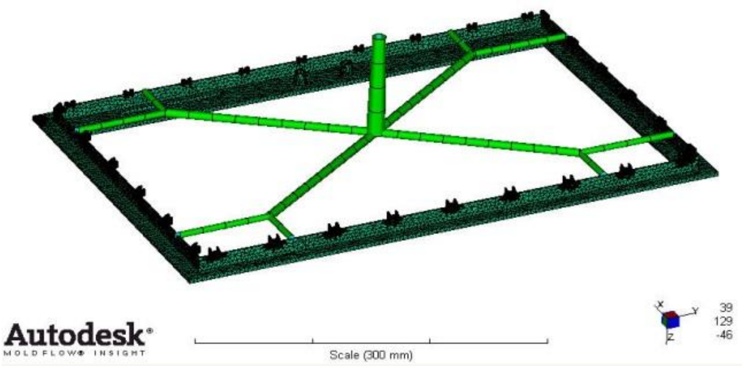

Figure 5. The modeled runner system in Mold Flow

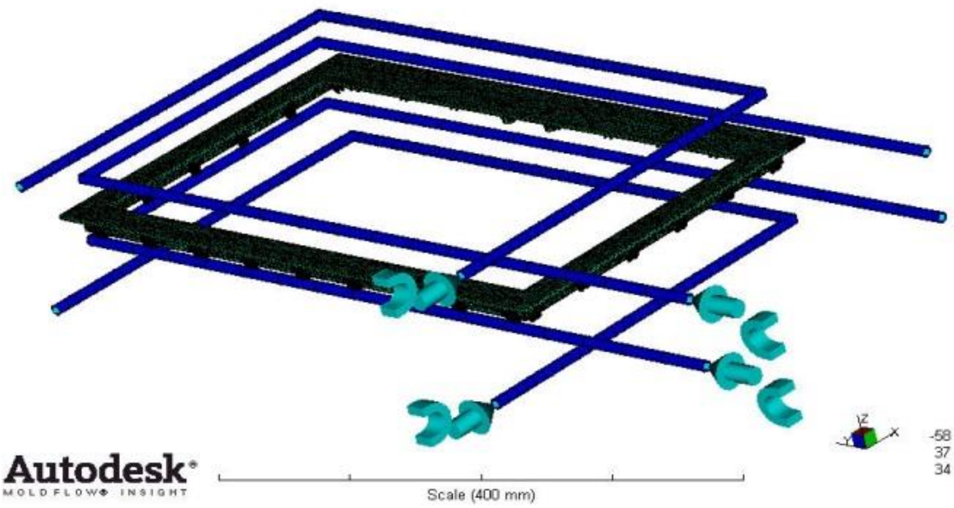

Figure 6. The modeled cooling circuit in Mold Flow software

After several investigations for the appropriate setting by using the simulation software, the most appropriate treatment is determined as $BT = 200\ ^\circ\mathrm{C}$, TRH =2.5s, PH=60%, VH=50% and pp=20%, which results the deflection rate = 6.53%, sink mark ratio = 1.792% and the volumetric shrinkage = 4.09%. The results are illustrated in Figures 7, 8 and 9.

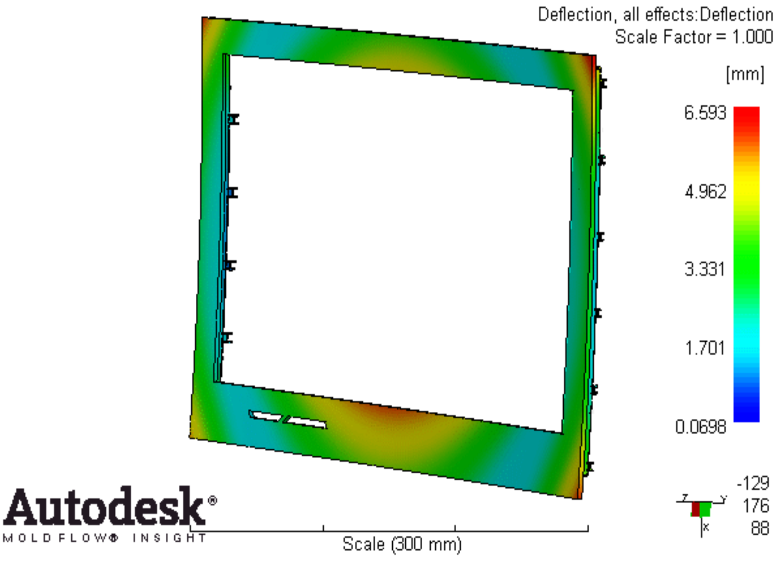

Figure 7. The result of deflection analysis

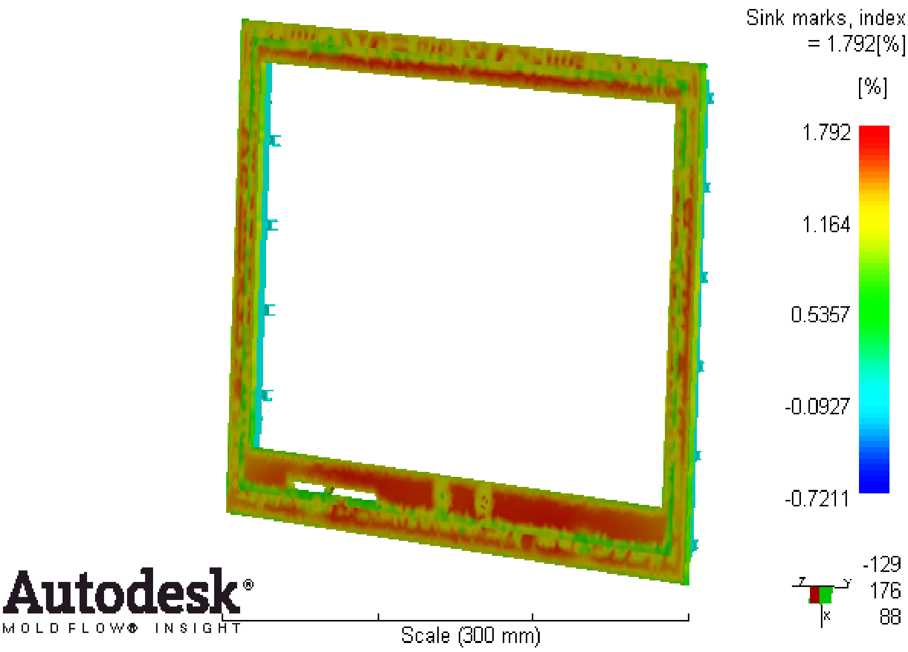

Figure 8. The result of sink mark analysis

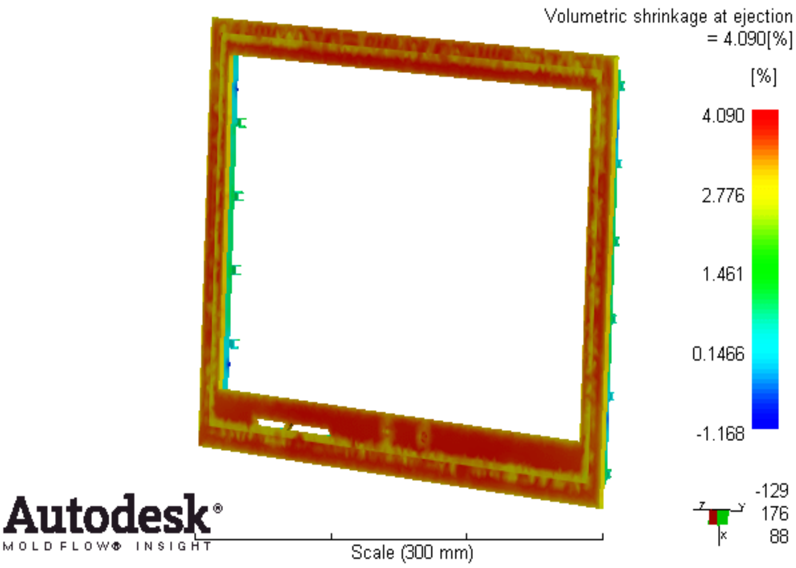

Figure 9. The result of volumetric shrinkage analysis

In these figures, the status of each property is illustrated on different zones of the part using a color scale. The spectrum from blue color to red indicates least value of the corresponding index to the most value of it on different zones of the part. For instance, consider figure 7 in which the maximum deflection equals to 6.593 *mm*, occurs in very small zones of the part on the four corners, while the rest zones have less deflection illustrated with green color or blue. Similarly, for the sink mark index the most value of the index is 1.792 *%*.

The homogeneity of sink mark index in most zones of the part shows that less sink mark points may occur on the part. Note that optimization of the sink mark index and the volumetric shrinkage in thin shell parts with large sizes, is a challenging procedure and needs a lot of investigation, for which some researchers have studied about the optimization of such cases, for instance see Guo et al. (2014), Chiang and Chang (2007).

The obtained result for the sink mark index is equal to 1.792% (the most value of sink mark index on the part) and the volumetric shrinkage percentage is equal to 4.09% (the most value of volumetric shrinkage on the part). These results are persuasive for the part under study. The simulation results are summarized in Table 7.

Table 7. The simulation results to find most appropriate settings for the part properties

The most appropriate setting based on simulation study	Deflection index	Sink mark index	Volumetric shrinkage index
$BT = 200$ ℃, TRH =2.5s, PH=60%, VH=50% pp=20%	6.593*mm*	1.792%	4.09%

The comparison between the optimum settings obtained from the proposed approach and the simulation studies shows that the proposed method leads to reliable results. These results are near optimum values for the responses under study, namely the number of sink marks and the size of the lower side. Moreover, the other properties of the part, such as the deflection index, sink mark index and the volumetric shrinkage reach desirable values as well. This comparison is presented in Table 8 as follows:

Table 8. The comparison of the obtained setting from the proposed method and the simulation

Approach	Optimum Setting
The setting obtained using the proposed approach	$BT = 200$ ℃, *TRH* =3s, *PH*=50%, *VH*=63.5%, *PP*=20%.
The setting obtained using the simulation approach	$BT = 200$ ℃, *TRH* =2.5s, *PH*=50%, *VH*=60%, *PP*=20%.

5. Conclusions

In this paper a new approach was proposed for a multi response optimization problem where the target values of responses were fuzzy numbers, and some responses followed non-normal distribution. In this method, after transformation of initial responses into normally distributed ones, the regression function of each response was written in terms of controllable factors. Then a fuzzy goal programming model proposed by Li and Hu (2009) was used and the most compromising values were determined for responses by solving this model. The computed results were tested in a verification experiment as well as a simulation study. The obtained results showed that not only the obtained optimum treatment results in appropriate values of the responses under study in practice, but also this treatment can result permissible plastic properties such as deflection, sink mark index and volumetric shrinkage. For future studies, it is recommended to apply other regression models for mixed continuous – discrete responses and to use the obtained regression functions as the objectives directly without the need to transform the non-normally distributed data.

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